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**Race:**
(Select one or more)
☐ American Indian or Alaska Native
☐ Asian
☐ Black or African American
☐ Native Hawaiian or Other Pacific Islander
☒ White

**Disability Status:**
(Select one or more)
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## List of Suggested Reviewers or Reviewers Not To Include (optional)

### SUGGESTED REVIEWERS:

**Suitable reviewers who come to mind are David Aldous, Joshua Cooper, Robert Ellis, Joel Spencer, Benjamin Doerr, and Gabor Tardos. It should be emphasized that although I use the word ‘‘quasirandom’’ in this research proposal, I do not use it in the same sense as Ronald Graham, Fan Chung, and Richard Wilson.**

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**Not Listed**

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# CERTIFICATION PAGE

## Certification for Authorized Organizational Representative or Individual Applicant:

By signing and submitting this proposal, the Authorized Organizational Representative or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding debarment and suspension, drug-free workplace, and lobbying activities (see below), nondiscrimination, and flood hazard insurance (when applicable) as set forth in the NSF Proposal & Award Policies & Procedures Guide, Part I: the Grant Proposal Guide (GPG) (NSF 09-29). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U. S. Code, Title 18, Section 1001).

### Conflict of Interest Certification

In addition, if the applicant institution employs more than fifty persons, by electronically signing the NSF Proposal Cover Sheet, the Authorized Organizational Representative of the applicant institution is certifying that the institution has implemented a written and enforced conflict of interest policy that is consistent with the provisions of the NSF Proposal & Award Policies & Procedures Guide, Part II, Award & Administration Guide (AAG) Chapter IV.A; that to the best of his/her knowledge, all financial disclosures required by that conflict of interest policy have been made; and that all identified conflicts of interest will have been satisfactorily managed, reduced or eliminated prior to the institution's expenditure of any funds under the award, in accordance with the institution's conflict of interest policy. Conflicts which cannot be satisfactorily managed, reduced or eliminated must be dislosed to NSF.

### Drug Free Work Place Certification

By electronically signing the NSF Proposal Cover Sheet, the Authorized Organizational Representative or Individual Applicant is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Grant Proposal Guide.

### Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

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### Certification Regarding Lobbying

The following certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

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The undersigned certifies, to the best of his or her knowledge and belief, that:

(1) No federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.

(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.

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This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

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(2) building (and any related equipment) is covered by adequate flood insurance.

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(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and
(2) for other NSF Grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

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* EAGER - EArly-concept Grants for Exploratory Research
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# Project Summary

The PI proposes to conduct research on quasirandom analogues of discrete random systems, specifically those that are constructed using the design principle of discrepancy minimization. Such deterministic systems can frequently be described in terms of simple components called rotor-routers. Despite the fact that systems made from rotor-routers are deterministic, they frequently capture many of the most salient features of the random systems they mimic. These deterministic systems can give us useful information about their random counterparts, but they are also worthy of study in their own right, with their unexpected levels of structure and their startling symmetries.

The objective of the project is to discover the major properties of such systems, especially those that relate in a direct way to the corresponding random systems. The work will employ techniques from combinatorics, algebra, and analysis, combined with computer-aided experimentation.

Part of the **intellectual merit** of the proposal lies in its foundational nature: the PI will be looking at some of the most basic ideas of probability theory, such as random walk, from a new perspective. This work will shed direct light on the relationship between local order and global order in spatially extended systems and may also shed light on the relationship between randomness and discrepancy. A further part of the intellectual merit of the proposal lies in its interdisciplinary character, with possibilities for transfer of ideas between probability, combinatorics, and computer science.

A **broader impact** of the proposed research is the inclusion of undergraduates as research assistants. The PI will tightly integrate education with research by training students in fundamental tools pertaining to combinatorics, probability, and discrete dynamical systems (both stochastic and deterministic) and then setting the trainees loose on unsolved problems. In so doing, the PI will develop the students' general skills in mathematical research, with the hope of encouraging many of them to become mathematicians or scientists, or just citizens with an appreciation of the nature of the scientific enterprise.

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For font size and page formatting specifications, see GPG section II.B.2.



*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

# Results from Prior NSF Support

During the past four years, I have used funding from the National Science Foundation's probability program, as part of an ongoing project (NSF award number 0644877) called "Quasirandomness in Discrete Probability Theory". This project was originally scheduled to last from 2006 to 2009; with a no-cost extension, it has been prolonged until 2010, with a total budget of $100,999.00. The following articles have been written so far in pursuance of that grant:

Chip-Firing and Rotor-Routing on Directed Graphs, by Alexander Holroyd, Lionel Levine, Karola Mészáros, Yuval Peres, James Propp and David Wilson (`arXiv:0801.3306`), published in *Progress in Probability* **60**, 331–364. This article lays some of the combinatorial groundwork of rotor-routing on finite directed graphs. In particular, it explains the dichotomy between transient and recurrent rotor-router configurations, and shows that the latter are equinumerous with recurrent chip-firing configurations. The article shows that the critical group, which appears in a natural way in connection with chip-firing, is also intimately connected with rotor-routing.

Rotor Walks and Markov Chains, by Alexander Holroyd and James Propp. (`arXiv:0904.4507`), to appear in the Proceedings of the AMS Special Sessions on Algorithmic Probability and Combinatorics (edited by Manuel Lladser, Robert Maier, Marni Mishna and Andrew Rechnitzer). This article shows that for various natural quantities associated with Markov chains (such as hitting probabilities, hitting times and stationary probabilities), derandomizations obtained by rotor-routing are governed by the same quantities, but with tighter concentration around the mean. E.g., for finite Markov chains, $n$ independent simulations of a random process have concentration $O(1/\sqrt{n})$, whereas $n$ stages of rotor-routing have concentration $O(1/n)$.

Discrete Low-Discrepancy Sequences, by Omer Angel, Alexander E. Holroyd, James B. Martin and James Propp, (`arXiv:0910.1077`), submitted to *Combinatorics, Probability and Computing*. This article fills a gap in an earlier version of the Holroyd-Propp article. The original prescription for rotor-routing required that the transition probabilities $P(x, y)$ in the Markov chain be rational numbers, and furthermore required that for each state $x$ of the Markov chain, the number of states $y$ for which $P(x, y) > 0$ must be finite. The Angel et al. article shows that these restriction can be lifted.

I also gave a half-dozen seminar talks on preliminary findings associated with ideas described in the Proposed Research section of this document.

# Proposed Research

In the past century a good deal of attention has been given to deterministic objects, and deterministic sequences of objects, that behave for many purposes as if they were random. There is a need for a flexible kind of probability theory whose theorems will draw their inspiration from traditional probability theory but will have different sorts of hypotheses, in which assumptions of randomness are replaced by assumptions about frequency and discrepancy. As I discuss in the section on Results from Prior NSF Support, I am currently laying the foundations for one such variation on probability theory, in which special emphasis is laid on those deterministic analogues of probabilistic models in which discrepancy is made as low as it can possibly be.

A typical instance of this theory is to irreducible Markov chains with two or more absorbing states. Suppose that, starting from a particular source state $s$, the probability of the Markov chain stopping in target state $t$ is $p$, so that if one did infinitely many independent trials, the frequency of absorption at $t$ in the first $N$ trials would almost surely converge to $p$ as $N \to \infty$. Holroyd and Propp [Holroyd and Propp, 2009] give a general recipe for derandomizing such a Markov chain via mechanisms called rotor-routers. For instance, in the case where a state $x$ in the Markov chain has two successors $y$ and $z$ (so that the transition probabilities $P(x, y)$ and $P(x, z)$ sum to 1), the natural sort of rotor to use would be one that guarantees that for all $N$, among the first $N$ times the chain was in state $x$, the number of times it went from $x$ to $y$ is the integer closest to $N$ times $P(x, y)$. Rotor-routers more generally control the discrepancy between the *observed* number of transitions from $x$ to $y$ (under non-random simulation) and the *expected* number of transitions from $x$ to $y$ (conditioned on the observed number of visits to $x$). It is not hard to show that, under mild hypotheses, the rotor-router analogue of the Markov chain exhibits the same absorption frequency $p$. More importantly, Holroyd and Propp showed that the rotor-router analogue of a Markov chain typically exhibits tighter concentration around $p$ than one would see in i.i.d. simulation. Note that if one does $N$ i.i.d. trials, one expects $Np \pm O(\sqrt{N})$ of the trials to result in the chain stopping in state $t$. Holroyd and Propp showed that, in many interesting examples, the number of absorptions at $t$ in $N$ rotor-router trials is $Np \pm O(1)$ (or $Np \pm O(\log N)$ in some cases). That is, the empirical estimate of $p$ obtained in $N$ quasirandom trials approximates $p$ with an error on the order of $1/N$ or $(\log N)/N$, rather than $O(1/\sqrt{N})$.

One reason to study rotor-routing analogues of stochastic processes is to discover the power and limitations of arguments based on discrepancy. The class of rotor-router mechanisms can be viewed as an endpoint in a continuum of models whose other, better-studied endpoint is the class of standard stochastic models, with the connecting link being the applicability of discrepancy-theoretic notions. In comparing the tightness of i.i.d. estimates $(1/\sqrt{N})$ with the tightness of rotor-router estimates ($1/N$ or $(\log N)/N$), one might be led to hope that the concentration phenomenon for rotor-router simulation could be useful in Monte Carlo studies, where the goal is to estimate $p$ and quantities like it. In this hope one might draw encouragement from the literature on Monte Carlo integration, in which the purely random method of estimating an integral can in many cases be improved upon by the choice of appropriate schemes, called "quasirandom" by Harald Niederreiter et al. [Niederreiter, 1992]. Furthermore, there are some instances in which the constructions of well-distributed sequences found in the quasirandomness literature coincide with constructions arising from derandomized Markov chains. Hence I have felt justified in appropriating the term "quasirandom" to apply to rotor-router analogues of random processes.

It would seem hard to beat the $O(1/N)$ error bound for an estimate based on $N$ trials, especially if such an estimate is required for every value of $N$. However, it appears that if one uses "balanced" rotors, one gets $N$-trial rotor-router estimates of $p$ with error that can be significantly less than $O(1/N)$. For finite Markov chains, if one uses balanced rotors, the discrepancy between $(\hat{p}_1 + \hat{p}_2 + \cdots + \hat{p}_N)/N$ and $p$ is $O(1/N^2)$, where $\hat{p}_k$ denotes the naive estimate for $p$ given by the outcome of $k$ (quasirandom) trials, that is to say, the number of successes in the first $k$ trials divided by $k$; and for the "goldbug" device described in [Kleber, 2005], which has infinitely many states, use of balanced rotors appears to reduce the discrepancy to $O(1/N^{4/3})$. An alternative approach to bringing the error below $O(1/N)$ is to carry out two different rotor-router simulations, using opposite directions of the rotors to cause cancellation of errors. Both approaches seem very promising and worthy of study.

Here is one example of a concrete conjecture that I intend to prove. Consider a finite directed graph with a rotor at each vertex that periodically cycles through some list of vertices of the graph. Pick a source vertex $s$ and two target vertices $t_1$ and $t_2$, and send a particle on a rotor-walk through the directed graph, using the rotors to decide where each new vertex should be sent, and sending the particle back to $s$ each time it reaches $t_1$ or $t_2$. For all $k \geq 1$, let $a_k$ be 1 (resp. 2) if, on the $k$th occasion of the particle arriving at a target, it arrives at $t_1$ (resp. $t_2$). I claim that the sequence $a_1, a_2, \ldots$ is periodic. This is not as easy to prove as one might think, since in fact the sequence of rotor-configurations that occur is *not* periodic; it is merely eventually periodic, and that is only enough to guarantee that the sequence $a_1, a_2, \ldots$ is eventually periodic. Likewise, the sequence of vertices visited by the particle is not in general periodic; one only sees periodic behavior if one looks at the successive visits to the targets. (This result may actually be proved soon, as I am working with MIT undergraduates Linda Zayas-Palmer and Giuliano Giacaglia who, under my supervision and Lionel Levine's, will attempt to solve this problem and some other related problems.)

A different direction in which I want to extend the concentration phenomenon is passing from quantities like $p$, which arise in the "linear" part of the theory of Markov chains, to quantities like the variance of the time until absorption. In my earliest explorations, I suspected that rotor-routing was only capable of capturing linear terms in the behavior of stochastic models. A few years ago, in unpublished work described in seminar talks (see e.g. pages 84 to 98 of `http://jamespropp.org/tour.pdf`), I showed that by using three species of particles instead of just the usual single species, and introducing appropriate dynamics for the particles (involving creation, propagation, and absorption), one can devise deterministic machines that concentrate around the variances and correlations of various random variables associated with Markov chains. I plan to write an article explaining this.

One important infrastructural issue concerns chip-firing and rotor-routing on infinite graphs. For chip-firing and rotor-routing on finite graphs, an important technical tool is the "abelian property", a principle asserting that the final state does not depend on the order in which certain operations are performed. Working out the details for infinite graphs would give us a better understanding of derandomization of Markov chains with infinitely many states. Also, building upon [Holroyd et al., 2008] by extending the definition of the critical group (see [Biggs, 1999] and [Cori and Rossin, 2000]) from finite directed graphs to infinite ones is of intrinsic interest, as it gives rise to a broad class of abelian groups.

It is fairly clear how one can straightforwardly extend the idea of rotor-routing to continuous-time processes in such a way as to yield analogues of the Holroyd-Propp theorems. More challenging, and potentially more useful, is the project of extending the rotor-routing to continuous-space processes. I have some preliminary ideas for how this might be done, and have in mind some numerical experiments I would like to perform, as a way of seeing whether the ideas have promise. If they do, I would try to experimentally identify the best approaches and then prove some concentration results.

I believe that some of the ideas described here will eventually find their uses in the Markov Chain Monte Carlo community. To that end, I plan to publish an article geared toward statisticians, taking some of the ideas of the Holroyd-Propp article and recasting our concentration results as quasi-random estimation results. For example, the article would state and prove a concentration theorem for estimating the expected value of a real-valued function of a Markov chain relative to the stationary measure of the chain. I will describe how, for some applications, rotor-router estimation of quantities gives a certainty interval rather than a confidence interval for the quantity of interest.

I will also treat the case in which rotor-routing is applied to Markov chains with randomized initial settings of the rotors, when the chain has the property that no state can be visited twice, and when there is an exit-state that the Markov chain must eventually enter. This situation serves as a good explanatory transition between standard Monte Carlo and rotor-router theory, because it fits in with the well-established idea of variance reduction. More specifically, if one applies rotor-routing to such a Markov chain using randomized initial settings of the rotors, re-starting the chain each time it hits the exit-state and thereby obtaining a sequence of runs from the start-state to the exit-state, each run viewed in isolation is distributed according to the law of the Markov chain; but the runs viewed jointly are far from independent, and indeed their mutual dependence is of a kind that reduces the variance of the quantities of interest. This general approach has a long history in the Monte Carlo literature, under the name "variance reduction"; however, I have not found any method of variance reduction that is based on the notion of discrepancy. Discrepancy-based variance reduction fits in with existing ideas in the Monte Carlo world, and it leads naturally to the rotor-router model, once one follows the discrepancy-idea to its natural conclusion and dispenses with the idea that each individual run needs to be a random draw from law of the Markov chain.

So far, the models that have been successfully derandomized with rotor-routers are stochastic models whose behavior can be analyzed exactly. The exact solvability of the random system is helpful when one is trying to show that a particular deterministic system has similar behavior. However, if rotor-routing is going to catch on among the simulation community, it will eventually have to find applications for problems that cannot be analyzed exactly. In such cases, one cannot make use of exact descriptions of the behavior of the random system in proving that the two systems behave similarly. Hence there is a need for theorems that allow one to say that if two systems have similar dynamics from a discrepancy-based point of view, then their behavior will be similar (as measured by discrepancy notions), without requiring advance knowledge of the behavior of either system. As a warm-up for stating and proving such theorems, I intend to take some of the Holroyd-Propp theorems and recast them in the foregoing vein.

Although the critical group of a finite graph can be abstractly characterized as the cokernel of the Laplacian, the concrete manifestation of the critical group in terms of recurrent chip-configurations is far from understood. Even the concrete manifestation of the identity element of the critical group presents baffling features that have defied attempts at analysis for extremely simple graphs, such as the $n$-by-$n$ square grid (see Figure 4 on page 9 of [Holroyd et al., 2008]).

One path around the current roadblock is provided by an insight due to Lionel Levine, who conjectured that if a graph $G$ is the intersection of the infinite square grid $\mathbf{Z}^2$ with the interior of a large circle in $\mathbf{R}^2$, the identity chip-configuration on $G$ should have 2 chips on nearly every site. David Wilson wrote code that validates Levine's belief. Clearly such graphs are among the first examples one should work on if one wants a theory of the structure of the chip-firing identity element on general graphs.

Extending Levine and Wilson's experiments with circles, I've looked at ellipses and noticed several intriguing phenomena. First, it appears that for an ellipse of aspect ratio $\sqrt{2}$ or $1/\sqrt{2}$, with axes parallel to the coordinate axes, the identity chip-configuration has 3 chips on nearly every site. Second, it appears that as the aspect ratio of an ellipse varies from $1/\sqrt{2}$ to 1, with the minor axis increasing as the major axis stays constant, the total number of chips stays remarkably constant. Thirdly, if one uses ellipses whose axes are at a 45 degree angle to the coordinate axes, it appears that for all values of the aspect ratio, the average number of chips per site is very close to 2. The file `http://jamespropp.org/ellipse-ident.doc` shows the relevant graphs, and the animations `http://jamespropp.org/ellipse.gif` and `http://jamespropp.org/tilt-ellipse.gif` show some of the interesting structures that arise. Figure 1 shows a typical still from these animations.

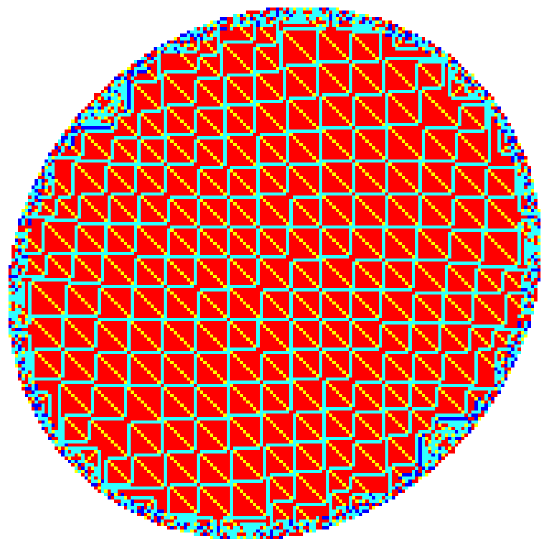

Figure 1: The identity element of the chip-firing group on an elliptical subgraph of $\mathbf{Z}^2$. The colors yellow, blue, red, and green represent sites with 0, 1, 2, and 3 chips, respectively.

Another problem I would like to tackle is the analysis of a fascinating fractal that Lionel Levine discovered in the context of rotor-routing in the plane. Figure 2 shows what happens if $N = 500$ indistinguishable particles are routed through the plane, where all rotors initially point to the East, with rotors turning counterclockwise. When a particle arrives at a site, the rotor at that site turns 90 degrees counterclockwise and the particle takes a step in the direction in which the rotor now points. The abelian property implies that the order in which we allow the particles to move does not affect the final configuration. Eventually all of the particles wander off towards infinity, never to return to the finite region shown. Each site is shown in one of four colors, according to the direction in which the rotor at that site points after it has been visited by the particles for the last time. This picture is remarkably stable as a function of the number of particles that have been

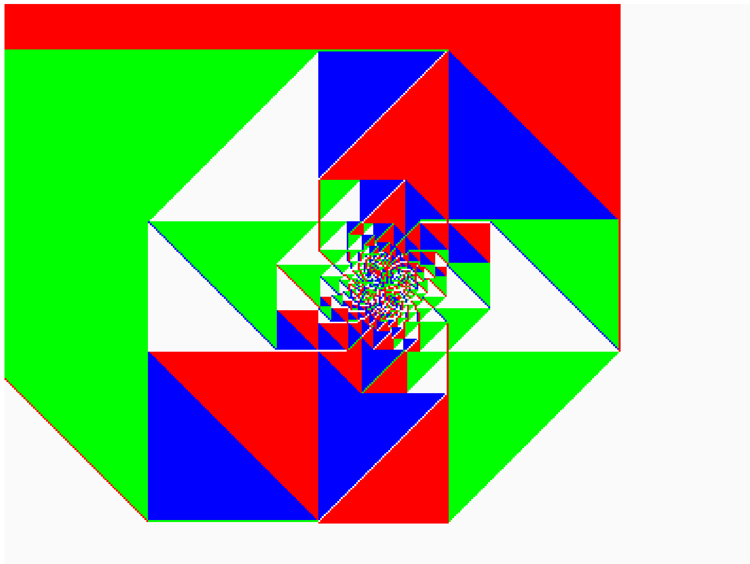

Figure 2: Rotor-routing in the plane, with all arrows initially aligned. The colors white, red, green, and blue represent sites with rotors pointing East, North, West, and South, respectively.

put through the system (if one allows for re-scaling), suggesting the existence of some sort of continuum limit. The dynamics that create and sustain the picture exhibit striking regularities and hints of periodic behavior, suggesting to me that it is reasonable to hope for a complete description of what one is seeing and a rigorous proof that the description is correct. (To view the video of which Figure 1 is a still, go to http://jamespropp.org/octagon.swf and click on the arrow in the middle of the screen.)

I also expect that I will be able to use the methods of the Holroyd-Propp article to establish a strong link between the behavior of this deterministic system (and its relatives) and the behavior of random walk in the plane. Suppose we put $N$ particles at the origin $(0,0)$, as above, but this time, any particle that arrives at $(1,1)$ is trapped there, and any particle that returns to $(0,0)$ is trapped there. If we let $A_N$, $B_N$, and $C_N$ denote the number of particles that wander off to infinity, get trapped at $(1,1)$, and get trapped at $(0,0)$, respectively, so that $A_N + B_N + C_N = N$, then the Holroyd-Propp theorems imply that $(\frac{1}{2}A_N + B_N)/N$ converges to the probability that a random walker who starts at $(0,0)$ reaches $(1,1)$ before returning to $(0,0)$, namely $\pi/8$. I hope to use the methods of the Holroyd-Propp article to show that $(\frac{1}{2}A_N + B_N)/N$ differs from $\pi/8$ by at most $O((\log N)/N)$. (Indeed, for $N \leq 1000$ we find that the difference is bounded by $2/N$, so it is possible that the numerator $\log N$ is overly pessimistic.) Interestingly, the rotor-state that results when all the particles have either been trapped or have left $[-n, n] \times [-n, n]$ for good is remarkably similar to Figure 2.

I also plan to return to the derandomized aggregation model I introduced to the mathematical community in [Kleber, 2005]. Levine and I and a few others (notably Matt Cook and Tobias Friedrich) have discovered many surprises in our simulations of rotor-router aggregation, some of which can be seen in detailed static pictures like the one at the bottom of Friedrich's homepage, and others of which only appear in dynamic simulations. It is time to write an article on some of these phenomena, both the ones we can prove and the (more numerous) ones we can't. One of the most striking is the observation that if we take the rotor-router aggregate scale it to fit in the unit circle in the complex plane, and apply the rational map $z \mapsto 1/z^2$, we find that the monochromatic patches are associated with points in a subgrid $\mathbf{Z} + i\mathbf{Z}$, as shown in Levine's picture. Another beautiful experimental finding is embodied in Matt Cook's plot that shows systematic behavior in the way in which the rotor-router aggregate deviates from circularity.

I came up with rotor-router aggregation in the late 1990s as a way of derandomizing internal diffusion limited aggregation, and Levine and Peres have made steady progress with understanding the relationship between the two models; see e.g. [Levine and Peres, 2007], which explains why the rotor-router aggregate is circular. Several years ago I came up with a similar deterministic model for interfaces between competing domains, except in this case it turned out that the corresponding stochastic model had escaped the attention of both physicists and probabilists. In both models, we imagine a graph (specifically, a graph obtained by intersecting the square grid with some large simply-connected domain in the plane) whose vertices are initially colored red and blue in some fashion, with two special vertices called the red and blue sources. (The red source is not required to be colored red, nor is the blue source required to be colored blue.) Assume that at the start there is at least one blue vertex. The red source emits a particle, and the particle walks through the graph until it hits a blue vertex; when this happens, the blue vertex turns red, and the particle disappears. Then the blue source emits a particle, and the particle walks through the graph until it hits a red vertex; when this happens, the red vertex turns blue, and the particle disappears. (In the stochastic model the particles perform ordinary random walk, while in the deterministic model the particles walk in accordance with rotors at the vertices.) This process repeats over and over, with the red source eroding the blue region and the blue source eroding the red region, so that eventually the vertices closer to the red source will tend to be red and the vertices closer to the blue source will tend to be blue.

Simulations show that the resulting interface between the red and blue vertices tends to be sharply defined, with fluctuations that appear to be on the order of the logarithm of the size of the domain, if not smaller. Moreover, it appears that the scaling limit of the interface reflects the conformal geometry of the domain in which the competition is occurring. To give the nicest example: when the domain is a disk, and the two competing sources are on the boundary of the disk, the interface that forms between the two domains within the disk will (conjecturally, in the limit) be a circular arc perpendicular to the boundary of the disk. More generally, given a simply-connected domain $D$ with smooth boundary, it appears that there is a conformal map from $D$ to the unit disk that sends the interface to a diameter of the disk. My prospective Ph.D. student David Einstein (currently enrolled in joint Ph.D. program sponsored by the computer science and mathematical sciences departments at UMass Lowell) has done simulations (using the method of coupling from the past; see [Propp and Wilson, 1996]) that support this conjecture. The most striking confirmation of the conjecture comes not from simulation of the random model itself, however, but from simulation of its deterministic rotor-router analogue. Here one sees interfaces that quickly snap into place, falling almost exactly where the conjectures about asymptotic behavior would lead one to expect to find them for the random case; see Einstein's simulation `http://jamespropp.org/qmdle.gif`. I intend to collaborate with Einstein, Levine and Peres and try to gain an understanding of both the random and derandomized versions of the mutual erosion model.

There are other spatial stochastic models for which I intend to seek deterministic analogues that exhibit the same gross behavior but with tighter concentration about the asymptotic shape. Specifically, I intend to look at percolation exploration, dimer models, and sorting networks.

My collaborator Ander Holroyd has done some preliminary work on derandomizing the percolation exploration process on a hexagonal grid using rotor-routers. He has found a simple deterministic mechanism that conforms to Cardy's formula for percolation on a triangular sub-region of the infinite hexagonal grid. That is, with Holroyd's dynamics, an exploration process that starts a one corner of the triangle is equally likely to leave via any of the possible exit-points along the opposite edge of the triangle. Moreover, Holroyd's process gives much tighter concentration around the uniform distribution on exit-points than one would get from an i.i.d. sequence of genuinely random exploration processes. Holroyd's mechanism is a type of rotor-routing mechanism, and his result requires the initial setting of the rotors to be rather special. It would be good to try other settings of the rotors to see if the Cardy's formula behavior still prevails. If it does, then one would like to see if, for other domains in the hexagonal grid, the deterministic percolation explorer gives the same exit probabilities (at least in the scaling limit) as one gets with the random percolation explorer.

Random states of dimer models have played a large role in my past research in combinatorics and probability, and I would like to revisit this topic from the viewpoint of quasirandomness. In particular, I would like to find a way to quasirandomize the domino shuffling algorithm of [Elkies et al., 1992]. This algorithm was developed as a way of sampling uniformly from the set of domino tilings of a region called the Aztec diamond of order $n$. For large values of $n$, a typical random tiling exhibits a change of phase as one passes from one side of a domain boundary to another, and this domain boundary, in the scaling limit, becomes the circle inscribed in the Aztec diamond (for more on the "Arctic Circle Theorem" see e.g. [Cohn, Elkies, and Propp, 1996]). There ought to be a non-random process that exhibits the same behavior in the large. I would also like to see if there is a good way to derandomize the heat bath Markov chain on the set of tilings. Here the approach of Holroyd and Propp is not readily applicable, since the Markov chain has far too many states. A method of derandomizing this class of model might lead to similar advances for other similar stat mech models (such as the square ice model).

Holroyd and others (see [Angel, Holroyd, Romik and Virag, 2006] have done some very interesting work on the behavior of random sorting networks, and in particular have shown that for large enough $n$, random sorting networks of order $n$ display a non-obvious kind of asymptotic geometry. I am hoping that a suitable deterministic scheme for generating a sequence of sorting networks will exhibit tight concentration around this asymptotic geometry. To study this, I intend to first come up with good (i.e. low-discrepancy) schemes for generating quasirandom Young tableaux, and then use the known bijection between Young tableaux and sorting networks.

In addition to writing articles on these topics, I plan to write a monograph outlining the current state of the theory of quasirandom analogues of stochastic processes, and, more importantly, indicating the governing philosophy. I find that even people with whom I have collaborated closely do not always see the connections that I have in mind. I think I ought to spell out these connections between combinatorics, probability, discrepancy theory, and computation, even if some of them are still speculative, so that people can see at least one sort of over-arching research program into which current research on these models can be fitted.

During all three years of the funded research, I intend to give my ideas a wide audience by writing articles in various journals and by giving talks at national and international conferences. Additionally, I will travel to work with collaborators at their home-institutions and arrange to have them visit me at UMass Lowell.

I will also stay in touch with other researchers who are exploring chip-firing and rotor-routing from different angles. Cooper, Doerr, Friedrich, Spencer, and Tardos have done important work showing that on certain graphs, multi-particle rotor-walk for a fixed number of steps has behavior that is extremely tightly concentrated around the average behavior of multi-particle random walk, aka discrete diffusion. (See [Cooper and Spencer, 2006], [Cooper et al., 2006], [Cooper et al., 2007], [Cooper et al., 2008], and [Doerr and Friedrich, 2009].) Physicists Chandra, Dhar, and Sadhu (see [Dhar, Sadhu and Chandra, 2008] and [Sadhu and Dhar, 2009]) analyzed the chip-firing model on a variant of the square grid that gives rise to an intriguing fractal shape reminiscent of (but different from) Figure 2; although their methods are not fully rigorous, their paper has ideas worth understanding. Similarly, the work of (A. and D.) Dhar, Krishnamurthy, Povolotsky, Priezzhev, and Shcherbakov (see [Priezzhev et al., 1996], [Povolotsky et al., 1998]) has many interesting ideas on rotor-router walks in the plane, which these authors invented (and dubbed "Eulerian walk") years before I came up with it independently. Friedrich, Gairing, and Sauerwald are looking into the use of rotor-routing for the design of quasirandom load-balancing schemes and algorithms for quasirandom rumor-spreading in networks (see [Friedrich, Gairing, and Sauerwald, 2010]). Cooper et al. (see [Cooper et al., 2009]) and Friedrich and Sauerwald (unpublished preprint) are also studying the cover-time of rotor-walk on graphs, building on earlier work of Yanovski [Yanovski et al., 2003] who independently invented rotor-walk ("ant walk") in the past decade. Lastly, Owen and Tribble have their own approach to quasirandom simulation of Markov chains (see [Owen and Tribble, 2005]), using the very natural and broad idea of completely uniform sequences, and it will be interesting to see the extent to which the two approaches to quasirandomness will overlap or will instead branch out in different directions.

My work has possible pedagogical implications. Quasirandom simulation might be a useful supplementary topic for the teaching of probability theory at a pre-college level. Arthur Engel invented his probabilistic abacus for the specific purpose of helping him teach probability theory to fourth-graders, and had some success with this (although this pilot effort never underwent assessment); he has written a book-length manuscript on the subject, and I am currently helping him locate a suitable publisher. Other researchers (under the auspices of DIMACS) have created their own teaching materials on chip-firing in the form of an educational module (see [Doty, Krog, and McGrail, 2004]). Calculating probabilities with chip-games is much more fun for children than solving simultaneous systems of linear equations. One potential drawback is that it is not possible to give children a rigorous yet age-appropriate answer to the question of why chip-games yield correct answers, and rotor-games are subject to the same liability; yet the same might be said for many algorithms taught in pre-college math, most of which are not nearly as gripping as quasirandom simulation. Also, rotor-mechanisms lead very naturally into the topic of radix representation, including non-integer bases, which high school students find exciting. (See the section on "Base One and a Half" halfway down the web-page `http://www.themathcircle.org/researchproblems.php`; this problem, which grew out of my work on chip-firing, was quite popular with the high students in the Boston Area Math Circle.)

Here are some activities related to my research that will significantly broaden its impact, above and beyond the publishing of research articles:
(1) I plan to write a very brief survey article with Lionel Levine for the *Notices of the American Mathematical Society*, describing what is known about chip-firing and the abelian sandpile model. (As I mentioned above, there is a very strong link between chip-firing and rotor-routers.) The existing literature is spread across various disciplines, and many researchers seem to be unaware of work done by people outside of their own field (and almost nobody knows about Engel's work). Some beautiful and challenging problems have not gotten as much attention from mathematicians as they deserve. I hope our article will help to remedy this.
(2) I plan to rewrite the unjustly neglected article of Faltin, Metropolis, Ross and Rota (Faltin et al., 1975). This article shows how one can construct the real numbers as decimals (or expansions using some other base, of course). Considering that this is the most prevalent way of representing real numbers,

it is in retrospect surprising that nobody thought to work out and streamline the details earlier. The key technical idea that makes the details pleasant (and not nearly as grotesque as one might predict) is an equivalence relation that turns out to be tantamount to a form of chip-firing. I plan to re-do this article in a way that makes the link with chip-firing explicit, and shows how many interesting variants of the standard radix-expansion idea can be obtained by varying the chip-firing rules. I will also explain how rotor-routers give an alternate (and in some ways superior) way to "reinvent the real numbers". I would submit this article to the American Mathematical Monthly. A preliminary version of these ideas will be presented at the 2010 "Gathering for Gardner" conference and subsequently published in a proceedings volume.

(3) I plan to correspond with Arthur Engel about his current efforts to create a curriculum and teacher's guide for the stochastic abacus. (So far all he has published is [Engel, 1975] and [Engel, 1976].)

(4) Lionel Levine and I intend to create an on-line museum of images and simulations related to chip-firing and rotor-routing. The final version of this will deal not only with random and quasirandom walk and aggregation models but also with representations of numbers, computation with numbers, and the links between these topics.

(5) I plan to run an undergraduate research group for three years. Some fundamental problems related to quasirandomness do not call for great mathematical sophistication, but rather for persistence, cleverness, and creativity. I have had a great deal of success in working with students at top schools in the Boston area, and am currently working with two students at MIT; I plan to continue to work with such students, as well as students at UMass Lowell. My research on quasirandomness offers a range of levels of difficulty, and also mixes different styles of research (ranging from the purely theoretical to the purely experimental), so I am confident that I will be able to give all my students a satisfying research experience that will also contribute to the growth of the field. As in the past, I will continue to aggressively recruit women participants. In response to student interest, I will offer training not just in the relevant mathematics but also in Mathematica and Maple (for running simulations and analyses) and LaTeX(for writing up results). Running this research group will take about twelve hours per week of my time for all six semesters. I am hoping that the outcome of this work will be half a dozen math arXiv preprints and two or three articles written by students or groups of students and published in professional journals.

(6) I plan to present talks to the Boston Math Circle, describing some of these ideas to elementary and high school students.

I am seeking support for this work from both the Algebra, Number Theory and Combinatorics program and the Probability program of NSF. Rotor-routing, like chip-firing, can be justified as a purely combinatorial enterprise, where simple discrete rules generate patterns that are complicated enough to command interest but not so complicated as to defeat solution. Quasirandom systems have elements of beauty, surprise, and hidden structure that make them intrinsically worthy of mathematical study, and instructive examples of the way in which simple local rules can lead to complex global behavior. However, purely combinatorial considerations will not guide us to the richest examples; I am convinced that the most beautiful and challenging examples in this subject will arise from derandomization of classical probabilistic constructions. At the same time, I expect that some combinatorial processes will turn out, after the fact, to be derandomized versions of probabilistic processes. In this way, quasirandomness based on minimization of discrepancy will be a two-way bridge between probability and combinatorics.

I expect that, as far as applied probability theory is concerned, the long-term outcome of this work will be similar to the outcome of my work with David Wilson on exact sampling: a half-dozen to a dozen researchers will take up the idea of quasirandom simulation and turn it into a useful tool for people in the sciences who use discrete stochastic systems as models and who need to assess how well their models fit reality. But I am hoping that the larger impact will be on pure probability theory, and that my work of discrepancy-based quasirandomness will lead others to conduct deeper work on "random" behavior in deterministic settings from as general a point of view as possible. Lastly, I hope that many of the undergraduates who assist me with this work will be inspired to pursue research careers of their own, and that the graduate students who help me supervise the research team will become enthusiastic about this model of research and spread it elsewhere.

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# Biography: James G. Propp

## (a) Professional Preparation

Harvard College, mathematics, A.B. 1982.

University of California at Berkeley, mathematics, Ph.D. 1987.

Postdoctoral work at University of Maryland (1987–1988) and University of California at Berkeley (1988–1990), ergodic theory, funded by NSF.

## (b) Appointments

University of Massachusetts Lowell, 2006 to present.

University of Wisconsin at Madison, 1998 to 2006.

Brandeis University, Visiting Associate Professor, 2002-2003

Harvard University, Visiting Associate Professor, 2001-2002

Olin College, Visiting Associate Professor, Fall 2000

Massachusetts Institute of Technology, Assistant Professor, 1990 to 1996; Associate Professor, 1996 to 1998; Visiting Scholar, 1998 to 2000.

## (c) Publications

Five publications most closely related to the proposal:

[Angel, Holroyd, Martin, and Propp, 2009]

[Holroyd and Propp, 2009]

[Holroyd, Levine, Mészáros, Peres, Propp, and Wilson, 2008]

[Kleber, 2005] (co-written with PI)

[Propp and Wilson, 1996].

Five other publications by the proposer:

[Propp, 2003]

[Cohn, Pemantle, and Propp, 2002]

[Cohn, Kenyon, and Propp, 2001]

[Propp and Wilson, 1998]

[Cohn, Elkies, and Propp, 1996].

(Full publication information on the above articles appears in the References Cited section of this proposal.)

## (d) Synergistic Activities

1. I have given a dozen talks on chip-firing and rotor-routing to high school and college students, with an emphasis on all-girl/woman institutions.

2. In 2008, David Wilson and I co-wrote a chapter for the book *Markov Chains and Mixing Times* by Levin, Peres, and Wilmer. Our chapter brings

the simulation method called coupling from the past (invented by us back in the 1990s) to a broader audience.

3. As a teacher, I have put curricular materials (lecture notes, problem sets) on the web so that they can be adopted by other teachers or be used by students for self-study. My on-line materials on (Algebraic Combinatorics) and (Stochastic Processes) have both been used for self-study by researchers.

4. I have organized vertically-integrated research communities of undergraduates and graduate students at MIT (the Tilings Research Group), the University of Wisconsin (the Spatial Systems Lab), and Harvard University (Research Experiences in Algebraic Combinatorics at Harvard).

5. I have been the moderator of the domino forum (an electronic community of mathematicians, computer scientists, and physicists, as well as graduate students and undergraduates) since 1995.

**(e) Collaborators and Other Affiliations**

Collaborators: Omer Angel (University of British Columbia), Mireille Bousquet-Mélou (Laboratoire Bordelais de Recherche en Informatique), Matt Cook (California Institute of Technology), David Feldman (University of New Hampshire), Tobias Friedrich (Max-Planck-Institut für Informatik), Alexander Holroyd (Microsoft Research), Lionel Levine (Massachusetts Institute of Technology), Stephen Linton (University of St. Andrews), James Martin (Oxford University), Karola Meszaros (Massachusetts Institute of Technology), Gregg Musiker (Massachusetts Institute of Technology), Yuval Peres (Microsoft Research), Sinai Robins (Nanyang Technological University and Temple University), Thomas Roby (University of Connecticut), Julian West (University of St. Andrews), and David Wilson (Microsoft Research).

Co-editors: I am nominally on the editorial boards of two journals: the *Online Journal of Analytic Combinatorics*, with managing editors Alex Iosevich (University of Missouri), Izabella Laba (University of British Columbia), Sinai Robins (Nanyang Technological University and Temple University), and *Involve, a Journal of Mathematics*, with managing editor Kenneth Berenhaut (Wake Forest University). I am not currently active on either board.

Graduate Advisor: Jacob Feldman, University of California at Berkeley (emeritus). Principal postdoctoral sponsors: N/A.

Thesis Advisor and Postgraduate-Scholar Sponsor: N/A.

# SUMMARY PROPOSAL BUDGET

YEAR 1

| ORGANIZATION | FOR NSF USE ONLY | | |
|---|---|---|---|
| University of Massachusetts Lowell | PROPOSAL NO. | DURATION (months) Proposed | Granted |
| PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR<br>James G Propp | AWARD NO. | | |

| A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets) | NSF Funded Person-months CAL | ACAD | SUMR | Funds Requested By proposer | Funds granted by NSF (if different) |
|---|---|---|---|---|---|
| 1. James G Propp - Principal Investigator | 0.00 | 3.00 | 1.00 | $ 49,749 | $ |
| 2. | | | | | |
| 3. | | | | | |
| 4. | | | | | |
| 5. | | | | | |
| 6. ( 0 ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE) | 0.00 | 0.00 | 0.00 | 0 | |
| 7. ( 1 ) TOTAL SENIOR PERSONNEL (1 - 6) | 0.00 | 3.00 | 1.00 | 49,749 | |
| B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS) | | | | | |
| 1. ( 0 ) POST DOCTORAL SCHOLARS | 0.00 | 0.00 | 0.00 | 0 | |
| 2. ( 0 ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.) | 0.00 | 0.00 | 0.00 | 0 | |
| 3. ( 1 ) GRADUATE STUDENTS | | | | 21,000 | |
| 4. ( 4 ) UNDERGRADUATE STUDENTS | | | | 20,800 | |
| 5. ( 0 ) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY) | | | | 0 | |
| 6. ( 0 ) OTHER | | | | 0 | |
| TOTAL SALARIES AND WAGES (A + B) | | | | 91,549 | |
| C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) | | | | 757 | |
| TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C) | | | | 92,306 | |
| D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) | | | | | |
| TOTAL EQUIPMENT | | | | 0 | |
| E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS) | | | | 1,500 | |
| 2. FOREIGN | | | | 1,500 | |
| F. PARTICIPANT SUPPORT COSTS<br>1. STIPENDS $ 0<br>2. TRAVEL 0<br>3. SUBSISTENCE 0<br>4. OTHER 0 | | | | | |
| TOTAL NUMBER OF PARTICIPANTS ( 0 ) TOTAL PARTICIPANT COSTS | | | | 0 | |
| G. OTHER DIRECT COSTS | | | | | |
| 1. MATERIALS AND SUPPLIES | | | | 0 | |
| 2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION | | | | 0 | |
| 3. CONSULTANT SERVICES | | | | 2,000 | |
| 4. COMPUTER SERVICES | | | | 0 | |
| 5. SUBAWARDS | | | | 0 | |
| 6. OTHER | | | | 3,000 | |
| TOTAL OTHER DIRECT COSTS | | | | 5,000 | |
| H. TOTAL DIRECT COSTS (A THROUGH G) | | | | 100,306 | |
| I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)<br>51% MTDC (Rate: 51.0000, Base: 97306) | | | | | |
| TOTAL INDIRECT COSTS (F&A) | | | | 49,626 | |
| J. TOTAL DIRECT AND INDIRECT COSTS (H + I) | | | | 149,932 | |
| K. RESIDUAL FUNDS | | | | 0 | |
| L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K) | | | | $ 149,932 | $ |

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

| PI/PD NAME | FOR NSF USE ONLY | | |
|---|---|---|---|
| James G Propp | INDIRECT COST RATE VERIFICATION | | |
| ORG. REP. NAME* | Date Checked | Date Of Rate Sheet | Initials - ORG |

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

# SUMMARY PROPOSAL BUDGET

YEAR 2

| | | | | | | |
|---|---|---|---|---|---|---|
| ORGANIZATION<br>**University of Massachusetts Lowell** | | | | FOR NSF USE ONLY<br>PROPOSAL NO. | DURATION (months)<br>Proposed | Granted |
| PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR<br>**James G Propp** | | | | AWARD NO. | | |
| A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets) | NSF Funded Person-months CAL | ACAD | SUMR | Funds Requested By proposer | Funds granted by NSF (if different) | |
| 1. **James G Propp - Principal Investigator** | 0.00 | 3.00 | 1.00 | $ **51,241** | $ | |
| 2. | | | | | | |
| 3. | | | | | | |
| 4. | | | | | | |
| 5. | | | | | | |
| 6. ( **0** ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE) | 0.00 | 0.00 | 0.00 | **0** | | |
| 7. ( **1** ) TOTAL SENIOR PERSONNEL (1 - 6) | 0.00 | 3.00 | 1.00 | **51,241** | | |
| B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS) | | | | | | |
| 1. ( **0** ) POST DOCTORAL SCHOLARS | 0.00 | 0.00 | 0.00 | **0** | | |
| 2. ( **0** ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.) | 0.00 | 0.00 | 0.00 | **0** | | |
| 3. ( **1** ) GRADUATE STUDENTS | | | | **21,630** | | |
| 4. ( **4** ) UNDERGRADUATE STUDENTS | | | | **21,424** | | |
| 5. ( **0** ) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY) | | | | **0** | | |
| 6. ( **0** ) OTHER | | | | **0** | | |
| TOTAL SALARIES AND WAGES (A + B) | | | | **94,295** | | |
| C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) | | | | **780** | | |
| TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C) | | | | **95,075** | | |
| D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) | | | | | | |
| TOTAL EQUIPMENT | | | | **0** | | |
| E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS) | | | | **1,500** | | |
| 2. FOREIGN | | | | **1,500** | | |
| F. PARTICIPANT SUPPORT COSTS<br>1. STIPENDS $ **0**<br>2. TRAVEL **0**<br>3. SUBSISTENCE **0**<br>4. OTHER **0** | | | | | | |
| TOTAL NUMBER OF PARTICIPANTS ( **0** ) TOTAL PARTICIPANT COSTS | | | | **0** | | |
| G. OTHER DIRECT COSTS | | | | | | |
| 1. MATERIALS AND SUPPLIES | | | | **0** | | |
| 2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION | | | | **0** | | |
| 3. CONSULTANT SERVICES | | | | **2,000** | | |
| 4. COMPUTER SERVICES | | | | **0** | | |
| 5. SUBAWARDS | | | | **0** | | |
| 6. OTHER | | | | **3,000** | | |
| TOTAL OTHER DIRECT COSTS | | | | **5,000** | | |
| H. TOTAL DIRECT COSTS (A THROUGH G) | | | | **103,075** | | |
| I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)<br>**51% MTDC (Rate: 51.0000, Base: 100075)** | | | | | | |
| TOTAL INDIRECT COSTS (F&A) | | | | **51,038** | | |
| J. TOTAL DIRECT AND INDIRECT COSTS (H + I) | | | | **154,113** | | |
| K. RESIDUAL FUNDS | | | | **0** | | |
| L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K) | | | | $ **154,113** | $ | |
| M. COST SHARING PROPOSED LEVEL $ **0** | AGREED LEVEL IF DIFFERENT $ | | | | | |
| PI/PD NAME<br>**James G Propp** | | | | FOR NSF USE ONLY<br>INDIRECT COST RATE VERIFICATION | | |
| ORG. REP. NAME* | | | | Date Checked | Date Of Rate Sheet | Initials - ORG |

 ***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

# SUMMARY PROPOSAL BUDGET

YEAR 3

| FOR NSF USE ONLY | | |
|---|---|---|
| PROPOSAL NO. | DURATION (months) | |
| | Proposed | Granted |
| AWARD NO. | | |

ORGANIZATION
**University of Massachusetts Lowell**

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
**James G Propp**

| A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets) | NSF Funded Person-months CAL | ACAD | SUMR | Funds Requested By proposer | Funds granted by NSF (if different) |
|---|---|---|---|---|---|
| 1. James G Propp - Principal Investigator | 0.00 | 3.00 | 1.00 | $ 51,626 | $ |
| 2. | | | | | |
| 3. | | | | | |
| 4. | | | | | |
| 5. | | | | | |
| 6. ( 0 ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE) | 0.00 | 0.00 | 0.00 | 0 | |
| 7. ( 1 ) TOTAL SENIOR PERSONNEL (1 - 6) | 0.00 | 3.00 | 1.00 | 51,626 | |
| B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS) | | | | | |
| 1. ( 0 ) POST DOCTORAL SCHOLARS | 0.00 | 0.00 | 0.00 | 0 | |
| 2. ( 0 ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.) | 0.00 | 0.00 | 0.00 | 0 | |
| 3. ( 1 ) GRADUATE STUDENTS | | | | 22,279 | |
| 4. ( 4 ) UNDERGRADUATE STUDENTS | | | | 22,067 | |
| 5. ( 0 ) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY) | | | | 0 | |
| 6. ( 0 ) OTHER | | | | 0 | |
| TOTAL SALARIES AND WAGES (A + B) | | | | 95,972 | |
| C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) | | | | 803 | |
| TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C) | | | | 96,775 | |
| D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) | | | | | |
| TOTAL EQUIPMENT | | | | 0 | |
| E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS) | | | | 1,500 | |
| 2. FOREIGN | | | | 1,500 | |
| F. PARTICIPANT SUPPORT COSTS | | | | | |
| 1. STIPENDS $ 0 | | | | | |
| 2. TRAVEL 0 | | | | | |
| 3. SUBSISTENCE 0 | | | | | |
| 4. OTHER 0 | | | | | |
| TOTAL NUMBER OF PARTICIPANTS ( 0 ) TOTAL PARTICIPANT COSTS | | | | 0 | |
| G. OTHER DIRECT COSTS | | | | | |
| 1. MATERIALS AND SUPPLIES | | | | 0 | |
| 2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION | | | | 0 | |
| 3. CONSULTANT SERVICES | | | | 2,000 | |
| 4. COMPUTER SERVICES | | | | 0 | |
| 5. SUBAWARDS | | | | 0 | |
| 6. OTHER | | | | 3,000 | |
| TOTAL OTHER DIRECT COSTS | | | | 5,000 | |
| H. TOTAL DIRECT COSTS (A THROUGH G) | | | | 104,775 | |
| I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) 51% MTDC (Rate: 51.0000, Base: 101774) | | | | | |
| TOTAL INDIRECT COSTS (F&A) | | | | 51,905 | |
| J. TOTAL DIRECT AND INDIRECT COSTS (H + I) | | | | 156,680 | |
| K. RESIDUAL FUNDS | | | | 0 | |
| L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K) | | | | $ 156,680 | $ |

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
**James G Propp**

ORG. REP. NAME*

| FOR NSF USE ONLY | | |
|---|---|---|
| INDIRECT COST RATE VERIFICATION | | |
| Date Checked | Date Of Rate Sheet | Initials - ORG |
| | | |

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

# SUMMARY PROPOSAL BUDGET

Cumulative

| FOR NSF USE ONLY | | |
|---|---|---|
| PROPOSAL NO. | DURATION (months) | |
| | Proposed | Granted |
| AWARD NO. | | |

ORGANIZATION
University of Massachusetts Lowell

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
James G Propp

| A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets) | NSF Funded Person-months CAL | ACAD | SUMR | Funds Requested By proposer | Funds granted by NSF (if different) |
|---|---|---|---|---|---|
| 1. James G Propp - Principal Investigator | 0.00 | 9.00 | 3.00 | $ 152,616 | $ |
| 2. | | | | | |
| 3. | | | | | |
| 4. | | | | | |
| 5. | | | | | |
| 6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE) | 0.00 | 0.00 | 0.00 | 0 | |
| 7. ( 1 ) TOTAL SENIOR PERSONNEL (1 - 6) | 0.00 | 9.00 | 3.00 | 152,616 | |
| B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS) | | | | | |
| 1. ( 0 ) POST DOCTORAL SCHOLARS | 0.00 | 0.00 | 0.00 | 0 | |
| 2. ( 0 ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.) | 0.00 | 0.00 | 0.00 | 0 | |
| 3. ( 3 ) GRADUATE STUDENTS | | | | 64,909 | |
| 4. ( 12 ) UNDERGRADUATE STUDENTS | | | | 64,291 | |
| 5. ( 0 ) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY) | | | | 0 | |
| 6. ( 0 ) OTHER | | | | 0 | |
| TOTAL SALARIES AND WAGES (A + B) | | | | 281,816 | |
| C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) | | | | 2,340 | |
| TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C) | | | | 284,156 | |
| D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) | | | | | |
| TOTAL EQUIPMENT | | | | 0 | |
| E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS) | | | | 4,500 | |
| 2. FOREIGN | | | | 4,500 | |
| F. PARTICIPANT SUPPORT COSTS<br>1. STIPENDS $ 0<br>2. TRAVEL 0<br>3. SUBSISTENCE 0<br>4. OTHER 0 | | | | | |
| TOTAL NUMBER OF PARTICIPANTS ( 0 ) TOTAL PARTICIPANT COSTS | | | | 0 | |
| G. OTHER DIRECT COSTS | | | | | |
| 1. MATERIALS AND SUPPLIES | | | | 0 | |
| 2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION | | | | 0 | |
| 3. CONSULTANT SERVICES | | | | 6,000 | |
| 4. COMPUTER SERVICES | | | | 0 | |
| 5. SUBAWARDS | | | | 0 | |
| 6. OTHER | | | | 9,000 | |
| TOTAL OTHER DIRECT COSTS | | | | 15,000 | |
| H. TOTAL DIRECT COSTS (A THROUGH G) | | | | 308,156 | |
| I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) | | | | | |
| TOTAL INDIRECT COSTS (F&A) | | | | 152,569 | |
| J. TOTAL DIRECT AND INDIRECT COSTS (H + I) | | | | 460,725 | |
| K. RESIDUAL FUNDS | | | | 0 | |
| L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K) | | | | $ 460,725 | $ |

M. COST SHARING PROPOSED LEVEL $ 0 AGREED LEVEL IF DIFFERENT $

PI/PD NAME
James G Propp

ORG. REP. NAME*

| FOR NSF USE ONLY | | |
|---|---|---|
| INDIRECT COST RATE VERIFICATION | | |
| Date Checked | Date Of Rate Sheet | Initials - ORG |

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

# Budget Justification

### A. Senior Personnel

Senior Personnel salary for Year 1 is for 1 month per summer at the PI's salary and a teaching buy-out at 33.3% of the PI's salary. Salary in Year 1 is assumed to be 3% above the PI's 2009–2010 salary, with a subsequent estimated increase of 3% from Year 1 to Year 2 and from Year 2 to Year 3.

- In Fall 2010, the PI will start an undergraduate research group that will operate from Fall 2010 through Spring 2013. In addition to teaching the students necessary rudiments of combinatorics and probability, with forays into abstract algebra and linear algebra and real analysis as may be required, the PI will help the students learn how to use computer algebra systems like Maple and Mathematica to conduct experiments, and how to present proofs of theorems and results of experiments in LaTeX. Most importantly, the students will learn the rhythm of research in mathematics, with its false starts, incremental gains, setbacks, and moments of sudden insight. The PI will also guide the students in creating a website that serves as a permanent record of their process of discovery.

- In Summer 2011, as well as Summer 2012, the PI will help the undergraduate research assistants write up the work they did during the two preceding semesters. Publishable results will be written up and submitted for publication. In cases where the paper falls short of publishability, the paper will be posted in the arXiv, so that it will become a permanent part of the research literature.

- In Summer 2013, the PI will help the undergraduate research assistants write up the work they did during the two preceding semesters. The PI will also do some final tidying up of the project, e.g., making sure that the web-site is complete and consistent, making sure that software created by the students is stable and well-documented, etc.

- In Years 1, 2 and 3, the PI will devote 12 hours per week to setting up and running an undergraduate research laboratory. The University of Massachusetts Lowell has no provisions for giving faculty teaching-credit for running a laboratory in a historically non-lab-based subject like mathematics, and the departmental teaching load (even for active researchers) is five courses per year; hence course release, to partially offset the PI's investment of time in running the laboratory, is appropriate. Given the PI's penchant for investing a great deal of effort into every course he teaches (as evidenced by his winning the UMass Lowell Mathematical Sciences Department Teaching Award in 2008), it is difficult for him to carry on research while teaching two courses at once, and nearly impossible while teaching three.

**B. Other Personnel**

The PI will hire two sorts of students under the grant:

- Graduate students (all three years): One 50% appointment for one semester each year with a 3% increase in years 2 and 3 of proposal. These graduate students would pursue research on more advanced topics, and would help supervise the URAs (see below).

- Undergraduate research assistants (URAs): 6 students in each year, paid at a student hourly rate ($12/hour), limited to $2,000 per student per year. These students would engage in collaborative research on problems whose solutions are not known in advance and which are germane to the PI's research on deterministic analogues of random processes. It might be appropriate to fund these students via an REU Supplement rather than pay them as technicians, given the nature of the work. Other students would focus on coding and would create high-performance software to be used as research tools and easy-to-use applets to spread the group's results via the World Wide Web.

**C. Fringe Benefits**

Fringe rates are as follows in year 1 with a .5% increase in each year of proposal: Faculty and Undergraduate Students, 1.38%; Graduate Research Assistant, 1.42%.

**E. Travel**

The PI requests coverage of transportation and subsistence for attendance and participation of Senior Personnel at scientific conferences in the U.S. and Canada, and collaboration with colleagues at other institutions. This request includes student travel as well. The PI also requests coverage for a small amount of foreign travel, to pay for trips to conference meetings in Europe or Asia during each year of proposal. For example, in summer 2010 the PI intends to attend the Institute for Elementary Studies in Canada and the Ninth International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing in Poland.

**G. Other Direct Costs**

**G.1. Materials and Supplies**

Major expenses are not anticipated.

**G.3. Consultant Services**

In each of the three years, the PI expects to invite collaborators to visit him in Massachusetts. The PI's two closest collaborators on this project are likely to be Ander Holroyd and Lionel Levine. Holroyd is at Microsoft Research Labs. Levine is currently a Moore Instructor at MIT, but will probably move elsewhere before the grant expires in 2013.

**G.4. Computer Services**

No special computer services are required, since the computer science department has kindly given me a guest account, and since the TeraGrid will be available to me for large-scale computations.

**G.6. Other** Funds in the amount of $3,000 per year are requested to cover a portion of the tuition/fees for the graduate student. The remainder is paid for by the University.

**I. Indirect costs**

The federally negotiated indirect cost rate is 51% of modified total direct costs excluding equipment and tuition.

# CURRENT & PENDING SUPPORT

**JAMES G. PROPP**

**<u>Pending</u>**

Project/Proposal Title: Deterministic Analogues of Random Processes
Source of Support: National Science Foundation
Total Award Amount: $460,725
Total Award Period Covered: 7/1/10 to 6/30/13
Location of Project: University of Massachusetts Lowell
Person-Months Per Year Committed to the Project. Cal: ___ Acad: ____ Sumr: __1.00_

**<u>Current</u>**

Project/Proposal Title: Quasirandomness in Discrete Probability Theory
Source of Support: National Science Foundation
Total Award Amount: $100,999
Total Award Period Covered: 6/15/06 to 6/30/10
Location of Project: University of Massachusetts Lowell
Person-Months Per Year Committed to the Project. Cal: ___ Acad: ____ Sumr: __1.00_

# FACILITIES

**FACILITIES:** Identify the facilities to be used at each performance site listed and, as appropriate, indicate their capacities, pertinent capabilities, relative proximity, and extent of availability to the project. Use "Other" to describe the facilities at any other performance sites listed and at sites for field studies. Use additional pages if necessary.

**Laboratory:** **N/A**

**Clinical:** **N/A**

**Animal:** **N/A**

**Computer:** **MacBook Pro**

**Office:** **N/A**

**Other:** **N/A**

**MAJOR EQUIPMENT:** List the most important items available for this project and, as appropriate, identify the location and pertinent capabilities of each.

**N/A**

**OTHER RESOURCES:** Provide any information describing the other resources available for the project. Identify support services such as consultant, secretarial, machine shop, and electronics shop, and the extent to which they will be available for the project. Include an explanation of any consortium/contractual/subaward arrangements with other organizations.

**N/A**