List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:
Not Listed

REVIEWERS NOT TO INCLUDE:
Not Listed

The following information regarding collaborators and other affiliations (COA) must be separately provided for each individual identified as senior project personnel. The COA information must be provided through use of this COA template.

Please complete this template (e.g., Excel, Google Sheets, LibreOffice), save as .xlsx or .xls, and upload directly as a Fastlane Collaborators and Other Affiliations single copy doc. Do not upload .pdf.

If there are more than 10 individuals designated as senior project personnel on the proposal, or if there are print preview issues, each completed template must be saved as a .txt file [select the Text (Tab Delimited) option] rather than as an .xlsx or .xls file. This format will still enable preservation of searchable text and avoid delays in processing and review of the proposal.
Please note that some information requested in prior versions of the PAPPG is no longer requested. THIS IS PURPOSEFUL AND WE NO LONGER REQUIRE THIS INFORMATION TO BE REPORTED. Certain relationships will be reported in other sections (i.e., the names of postdoctoral scholar sponsors should not be reported, however if the individual collaborated on research with their postdoctoral scholar sponsor, then they would be reported as a collaborator). The information in the tables is not required to be sorted, alphabetically or otherwise.

There are five separate categories of information which correspond to the five tables in the COA template:

COA template Table 1:
List the individual's last name, first name, middle initial, and organizational affiliation (including considered affiliation) in the last 12 months.

COA template Table 2:
List names as last name, first name, middle initial, for whom a personal, family, or business relationship would otherwise preclude their service as a reviewer.

COA template Table 3:
List names as last name, first name, middle initial, and provide organizational affiliations, if known, for the following:
- **The individual's Ph.D. advisors; and**
- **All of the individual's Ph.D. thesis advisees.**

COA template Table 4:
List names as last name, first name, middle initial, and provide organizational affiliations, if known, for the following:
- **Co-authors on any book, article, report, abstract or paper with collaboration in the last 48 months (publication date may be later); and**
- **Collaborators on projects, such as funded grants, graduate research or others in the last 48 months.**

COA template Table 5:
List editorial board, editor-in chief and co-editors with whom the individual interacts. An editor-in-chief must list the entire editorial board.
- **Editorial Board: List name(s) of editor-in-chief and journal in the past 24 months; and**
- **Other co-Editors of journal or collections with whom the individual has directly interacted in the last 24 months.**

The template has been developed to be fillable, however, the content and format requirements must not be altered by the user. This template must be saved in .xlsx or .xls format, and directly uploaded into FastLane as a Collaborators and Other Affiliations Single Copy Document. Using the .xlsx or .xls format will enable preservation of searchable text that otherwise would be lost. It is therefore imperative that this document be uploaded in .xlsx or .xls only. Uploading a document in any format other than .xlsx or .xls may delay the timely processing and review of the proposal.

This information is used to manage reviewer selection. See Exhibit II-2 for additional information on potential reviewer conflicts.

1 Note that graduate advisors are no longer required to be reported.
2 Editorial Board does not include Editorial Advisory Board, International Advisory Board, Scientific Editorial Board, or any other subcategory of Editorial Board. It is limited to those individuals who perform editing duties or manage the editing process (i.e., editor in chief).

List names as Last Name, First Name, Middle Initial. Additionally, provide email, organization, and department (optional)
Fixed column widths keep this sheet one page wide; if you cut and paste text, set font size at 10pt or smaller, and
To insert *n* blank rows, select *n* row numbers to move down, right click, and choose Insert from the menu.
You may fill-down (crtl-D) to mark a sequence of collaborators, or copy affiliations. Excel has arrows that enable sorting.
For "Last Active Date" and "Last Active" columns dates are optional, but will help NSF staff easily determine which information remains relevant for reviewer selection.
"Last Active Date" and "Last Active" columns may be left blank for ongoing or current affiliations.

Table 1: List the individual's last name, first name, middle initial, and organizational affiliation (including considered affiliation) in the last 12 months.

1	Your Name:	Your Organizational Affiliation(s), last 12 r	Last Active Date
	Hulse, Thomas A.	Morgan State University, Assistant Professor	
		Colby College, Visiting Assistant Professor	8/31/2017

Table 2: List names as last name, first name, middle initial, for whom a personal, family, or business relationship would otherwise preclude their service as a reviewer.

R: Additional names for whom some relationship would otherwise preclude their service as a reviewer.

to disambiguate common names

2	Name:	Organizational Affiliation	Optional (email, Department)	Last Active
R:	Korman, Joanna	Naval Research Laboratory		
R:	Korman, Neil J.	Case Western Reserve University		
R:	Korman, Benjamin D.	University of Rochester		

Table 3: List names as last name, first name, middle initial, and provide organizational affiliations, if known, for the following.

G: The individual's Ph.D. advisors; and
T: All of the individual's Ph.D. thesis advisees.

to disambiguate common names

3	Advisor/Advisee Name:	Organizational Affiliation	Optional (email, Department)
G:	Hoffstein, Jeffrey E.	Brown University	jhoff@math.brown.edu

T:			

Table 4: List names as last name, first name, middle initial, and provide organizational affiliations, if known, for the following:

A: Co-authors on any book, article, report, abstract or paper with collaboration in the last 48 months (publication date may be later); and

C: Collaborators on projects, such as funded grants, graduate research or others in the last 48 months.

to disambiguate common names

4	Name:	Organizational Affiliation	Optional (email, Department)	Last Active
A:	Hoffstein, Jeffrey E.	Brown University	jhoff@math.brown.edu	
A:	Reznikov, Andre	Bar-Ilan University	reznikov@math.biu.ac.il	4/1/17
A:	Murty, Maruti R.	Queen's University	murty@mast.queensu.ca	12/14/16
A:	Kiral, Eren M.	Nesin Maths Village	erenmehmetkiral@protonmail.com	
A:	Kuan, Chan I.	Sun Yat-Sen University (Zhuhai)	kuanchi3@mail.sysu.edu.cn	
A:	Walker, Alexander W.	Brown University	alexander_walker@brown.edu	
A:	Lowry-Duda, David J.	University of Warwick	d.lowry@warwick.ac.uk	
A:	Tanabe, Naomi	Bowdoin College	ntanabe@bowdoin.edu	
A:	Yang, Jianing	Colby College	jyang@colby.edu	
A:	Lim, Li-Mei	Boston University	mei121@bu.edu	

Table 5: List editorial board, editor-in chief and co-editors with whom the individual interacts. An editor-in-chief must list the entire editorial board.

B: Editorial Board: List name(s) of editor-in-chief and journal in the past 24 months; and

E: Other co-Editors of journal or collections with whom the individual has directly interacted in the last 24 months.

to disambiguate common names

5	Name:	Organizational Affiliation	Journal/Collection	Last Active
E:	Miller, Steven J.	Williams College	Journal of Number Theory	
B:	N'Guerekata, Gaston M	Morgan State University	International Journal of Evolution Equations	

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./DUE DATE
NSF 18-522 **03/01/18**

☐ Special Exception to Deadline Date Policy

FOR NSF USE ONLY
NSF PROPOSAL NUMBER

FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.)
HRD - HIST BLACK COLLEGES AND UNIV, (continued)

DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
				879941318	

EMPLOYER IDENTIFICATION NUMBER (EIN) OR TAXPAYER IDENTIFICATION NUMBER (TIN)
526002033

SHOW PREVIOUS AWARD NO. IF THIS IS
☐ A RENEWAL
☐ AN ACCOMPLISHMENT-BASED RENEWAL

IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY? YES ☐ NO ☒ IF YES, LIST ACRONYM(S)

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE
Morgan State University

AWARDEE ORGANIZATION CODE (IF KNOWN)
0020834000

ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE
Morgan State University
1700 East Cold Spring Lane
Baltimore, MD. 212510002

NAME OF PRIMARY PLACE OF PERF
Morgan State University

ADDRESS OF PRIMARY PLACE OF PERF, INCLUDING 9 DIGIT ZIP CODE
Morgan State University
1700 E Cold Spring Lane
Baltimore ,MD ,212510002 ,US.

IS AWARDEE ORGANIZATION (Check All That Apply) ☐ SMALL BUSINESS ☐ MINORITY BUSINESS ☐ FOR-PROFIT ORGANIZATION ☐ WOMAN-OWNED BUSINESS

☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **Excellence in Research: Shifted Convolution Sums for The Congruent Number Problem**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ **208,480**	**36** months	**07/01/18**	

THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW
☒ BEGINNING INVESTIGATOR
☐ DISCLOSURE OF LOBBYING ACTIVITIES
☐ PROPRIETARY & PRIVILEGED INFORMATION
☐ HISTORIC PLACES
☐ VERTEBRATE ANIMALS IACUC App. Date _____
PHS Animal Welfare Assurance Number _____
☒ TYPE OF PROPOSAL **Research**

☐ HUMAN SUBJECTS Human Subjects Assurance Number _____
Exemption Subsection _____ or IRB App. Date _____
☒ INTERNATIONAL ACTIVITIES: COUNTRY/COUNTRIES INVOLVED
CA UK CH
☒ COLLABORATIVE STATUS
Not a collaborative proposal

PI/PD DEPARTMENT
Mathematics

PI/PD FAX NUMBER
443-885-8216

PI/PD POSTAL ADDRESS
1700 East Cold Spring Lane
Baltimore, MD 212510002
United States

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Email Address
PI/PD NAME **Thomas A Hulse**	**DPhil**	**2013**	**443-885-1009**	**thomas.hulse@morgan.edu**
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

CERTIFICATION PAGE

Certification for Authorized Organizational Representative (or Equivalent) or Individual Applicant

By electronically signing and submitting this proposal, the Authorized Organizational Representative (AOR) or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding conflict of interest (when applicable), drug-free workplace, debarment and suspension, lobbying activities (see below), nondiscrimination, flood hazard insurance (when applicable), responsible conduct of research, organizational support, Federal tax obligations, unpaid Federal tax liability, and criminal convictions as set forth in the NSF Proposal & Award Policies & Procedures Guide (PAPPG). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U.S. Code, Title 18, Section 1001).

Certification Regarding Conflict of Interest

The AOR is required to complete certifications stating that the organization has implemented and is enforcing a written policy on conflicts of interest (COI), consistent with the provisions of PAPPG Chapter IX.A.; that, to the best of his/her knowledge, all financial disclosures required by the conflict of interest policy were made; and that conflicts of interest, if any, were, or prior to the organization's expenditure of any funds under the award, will be, satisfactorily managed, reduced or eliminated in accordance with the organization's conflict of interest policy. Conflicts that cannot be satisfactorily managed, reduced or eliminated and research that proceeds without the imposition of conditions or restrictions when a conflict of interest exists, must be disclosed to NSF via use of the Notifications and Requests Module in FastLane.

Drug Free Work Place Certification

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent), is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Proposal & Award Policies & Procedures Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant is providing the Debarment and Suspension Certification contained in Exhibit II-4 of the Proposal & Award Policies & Procedures Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:
(1) No Federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any Federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.
(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.
(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

Certification Regarding Nondiscrimination

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is providing the Certification Regarding Nondiscrimination contained in Exhibit II-6 of the Proposal & Award Policies & Procedures Guide.

Certification Regarding Flood Hazard Insurance

Two sections of the National Flood Insurance Act of 1968 (42 USC §4012a and §4106) bar Federal agencies from giving financial assistance for acquisition or construction purposes in any area identified by the Federal Emergency Management Agency (FEMA) as having special flood hazards unless the:
(1) community in which that area is located participates in the national flood insurance program; and
(2) building (and any related equipment) is covered by adequate flood insurance.

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant located in FEMA-designated special flood hazard areas is certifying that adequate flood insurance has been or will be obtained in the following situations:
(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and
(2) for other NSF grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

Certification Regarding Responsible Conduct of Research (RCR) (This certification is not applicable to proposals for conferences, symposia, and workshops.)

By electronically signing the Certification Pages, the Authorized Organizational Representative is certifying that, in accordance with the NSF Proposal & Award Policies & Procedures Guide, Chapter IX.B. , the institution has a plan in place to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduates, graduate students and postdoctoral researchers who will be supported by NSF to conduct research. The AOR shall require that the language of this certification be included in any award documents for all subawards at all tiers.

CERTIFICATION PAGE - CONTINUED

Certification Regarding Organizational Support

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that there is organizational support for the proposal as required by Section 526 of the America COMPETES Reauthorization Act of 2010. This support extends to the portion of the proposal developed to satisfy the Broader Impacts Review Criterion as well as the Intellectual Merit Review Criterion, and any additional review criteria specified in the solicitation. Organizational support will be made available, as described in the proposal, in order to address the broader impacts and intellectual merit activities to be undertaken.

Certification Regarding Federal Tax Obligations

When the proposal exceeds $5,000,000, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal tax obligations. By electronically signing the Certification pages, the Authorized Organizational Representative is certifying that, to the best of their knowledge and belief, the proposing organization:
(1) has filed all Federal tax returns required during the three years preceding this certification;
(2) has not been convicted of a criminal offense under the Internal Revenue Code of 1986; and
(3) has not, more than 90 days prior to this certification, been notified of any unpaid Federal tax assessment for which the liability remains unsatisfied, unless the assessment is the subject of an installment agreement or offer in compromise that has been approved by the Internal Revenue Service and is not in default, or the assessment is the subject of a non-frivolous administrative or judicial proceeding.

Certification Regarding Unpaid Federal Tax Liability

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal Tax Liability:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has no unpaid Federal tax liability that has been assessed, for which all judicial and administrative remedies have been exhausted or lapsed, and that is not being paid in a timely manner pursuant to an agreement with the authority responsible for collecting the tax liability.

Certification Regarding Criminal Convictions

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Criminal Convictions:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has not been convicted of a felony criminal violation under any Federal law within the 24 months preceding the date on which the certification is signed.

Certification Dual Use Research of Concern

By electronically signing the certification pages, the Authorized Organizational Representative is certifying that the organization will be or is in compliance with all aspects of the United States Government Policy for Institutional Oversight of Life Sciences Dual Use Research of Concern.

AUTHORIZED ORGANIZATIONAL REPRESENTATIVE	SIGNATURE	DATE
NAME		

TELEPHONE NUMBER	EMAIL ADDRESS	FAX NUMBER

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) - continued from page 1
(Indicate the most specific unit known, i.e. program, division, etc.)

DMS - ALGEBRA AND NUMBER THEORY

PROJECT SUMMARY

Overview:

The Congruent Number Problem asks for a description of all congruent numbers: numbers that are the areas of right triangles with rational-length sides. Though versions of this problem date back over a thousand years, it is presently understood as asking whether there exists a terminating algorithm that can determine if a number is or is not congruent.

The proposed PI for this project is Dr. Thomas A. Hulse, an Assistant Professor of Mathematics at Morgan State University, an HBCU. It has recently been discovered by Dr. Hulse and his collaborators that this problem is closely related to the asymptotic behavior of new families of shifted convolution sums of Fourier coefficients of theta functions. It is proposed that Dr. Hulse study these sums using spectral decomposition, regularization, and averaging tools that have been employed by his group very successfully in the recent past on other relevant projects. He intends to shed new light on this problem using his expertise and these novel techniques in work with his collaborators. Furthermore, Dr. Hulse plans to supervise undergraduate and graduate research assistants in this project, in the process training them in number theory and computational methods.

Intellectual Merit:

Tunnell's Theorem, proven by Jerrold B. Tunnell in 1983, provides a terminating criterion for the Congruent Number Problem on the condition that a special case of the Birch and Swinnerton-Dyer Conjecture, one of the remaining outstanding Millennium Prize Problems, holds true. As this case has not yet been proven, Tunnell's Theorem can only confirm with certainty that some integers are not congruent.

What is novel about the plan in this proposal is that, unlike Tunnell's Theorem, it does not require an investigation of elliptic curves, their corresponding automorphic forms, nor the Birch-Swinnerton-Dyer Conjecture. It seems to be an independent approach to the problem which might sidestep these issues altogether and, used in concert with Tunnell's Theorem, could provide a special case of the Birch and Swinnerton-Dyer Conjecture and so say when elements of a family of elliptic curves have nonzero rank.

Broader Impacts:

As the Congruent Number Problem and the Birch and Swinnerton-Dyer Conjectures are both famous problems in mathematics, any progress toward them would help elevate the interest and enthusiasm in mathematics and number theory at Morgan State University, an HBCU in the Baltimore region. Dr. Hulse intends to present details on this and other unsolved problems in number theory as part of a new regular seminar at Morgan State. He will also recruit Morgan students to participate in the research and guide them toward academic and professional success. Promoting study of number theory and its applications, such as to the field of cryptography, will in turn positively impact government and industry by preparing individuals with skills well-suited for careers in national security. The success of these research assistantships will also contribute to elevating the prominence and success of underrepresented groups in STEM. All results will be made public and submitted for publication in peer-reviewed journals and communicated at colloquia, conferences and elsewhere.

TABLE OF CONTENTS

For font size and page formatting specifications, see PAPPG section II.B.2.

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

Project Description

Project Outline

The proposed research focus of this project is the study of a new mathematical object, a shifted sum, whose properties are closely tied to The Congruent Number Problem, a centuries-old question with origins in the works of Diophantus of Alexandria [8]. The primary goal of this proposal is to make a significant contribution to the collective understanding of this problem while also recruiting undergraduate and graduate students to assist in the research process and so learn about analytic and computational number theory. In so doing, this project would keep the mission of Morgan State University (MSU), an Historically Black College and University, to offer *"innovative, inclusive, and distinctive educational experiences to a broad cross section of the population*," and so help elevate the participation and success of underrepresented groups in STEM fields.

This grant proposal requests salary support during the summer months for the Principal Investigator, Dr. Thomas A. Hulse, as well as stipend support for undergraduate and graduate student research assistants so that Dr. Hulse will be able to pursue work with his collaborators on The Congruent Number Problem and supervise relevant student research. All new results would be disseminated through publication to peer-reviewed journals as well as presentations at conferences, workshops, and colloquia.

The PI and his collaborators have a long and established history of research collaboration and will continue work on this research topic. Thus, the collaborators are not seeking any funding for their support of this proposal and their contribution to this project will not interfere with their other professional responsibilities.

Overall Research Goals

Like many famous problems in number theory, The Congruent Number Problem is incredibly old, is closely tied to deep unsolved conjectures in mathematics and has the virtue of

being easy to explain to a lay-person. A *congruent number*, t, is an integer that is the area of a rational right triangle, a right triangle with rational-length sides. One can show that the numbers 5, 6, and 7 are each congruent numbers, as illustrated in Fig. 1.

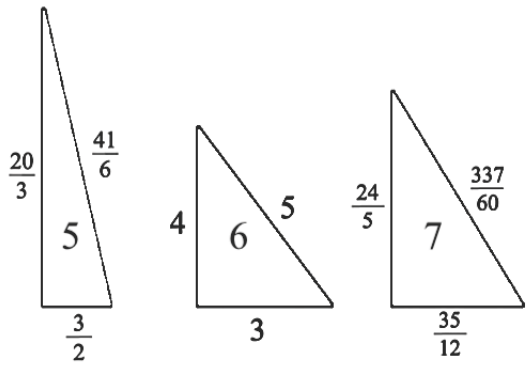

FIGURE 1. Rational right triangles with areas 5, 6 and 7.

Alternately, the numbers 1, 2, 3, and 4 are proven *not* to be congruent as a consequence of Fermat's right triangle theorem, proven in the mid-1600s and the only complete proof left by Pierre de Fermat [9]. So it is natural to ask which integers are, and which are not, congruent numbers.

The Congruent Number Problem is, specifically, the problem of finding a criterion that is proven to determine if a given t is, or is not, a congruent number in a computable amount of time. In 1983, Jerrold B. Tunnell was able to establish such a criterion [21], but the complete proof of its effectiveness depends on the truth of a special case of the unproven Birch and Swinnerton-Dyer Conjecture, one of the six yet unproven problems elevated by the Clay Mathematics Institute as the "Millennium Prize Problems," each endowed with a million-dollar bounty [6]. Tunnell used the correspondence between rational right triangles with area t and rational points on the elliptic curve $E : y^2 = x^3 - t^2x$, which in turn corresponds to coefficients of a known automorphic form by the theorems of Shimura, Tunnell, and Waldspurger [18].

It has recently been discovered by Principal Investigator Thomas Hulse and his collaborators, Chan Ieong Kuan, David Lowry-Duda and Alexander Walker, that the asymptotic properties of a certain family of shifted partial sums, parametrized by t, also determine if t is a congruent number. It remains to be proven that these properties can be computed by a terminating algorithm, and so investigating that is the main research goal of this project. These partial sums can be decomposed into smaller shifted sums of the kind that Dr. Hulse and his collaborators have a great deal of expertise investigating by means of shifted multiple Dirichlet series of automorphic forms, having done so to derive subconvexity bounds for automorphic L-functions [10], asymptotics of square discriminants [11], discerning the properties of partial sums of Fourier coefficients of automorphic forms, [14, 15, 16], and analyzing moments of the error term in the Gauss Circle Problem and its generalizations [13, 12]. As the methods of shifted sums have been generally agnostic about the properties of elliptic curves, Dr. Hulse is confident that this novel approach will yield a new, yet-unstudied avenue of pursuit for the congruent number problem, namely spectral decompositions of Dirichlet series, and will shed light on the relationship between the singularities of these analytic objects and pythagorean triples. The specific details of the objects being studied and the outline of the research program can be found in the section on Intellectual Merit.

Student Participation

As a new faculty member at MSU who believes strongly in the social mission of the school, the educational goals of this project are intended to complement and support the work that Dr. Hulse began as soon as he arrived in Baltimore. In the past year, Dr. Hulse has been chairing the Mathematical Competitions Committee in the MSU math department, in which he organized weekly sessions for students to practice competition-style problems to train for mathematical contests. This was the first year in which Morgan students participated in the Virginia Tech Regional Mathematics Competition [4] and the William Lowell Putnam Mathematical Competition [2], and in the later contest fielded a student who placed in the top 37% of 4,638 students. These weekly sessions have already been successful at enticing the interest of students in other STEM fields such as physics, engineering and computer science, and Dr. Hulse is continuing to work to grow the interdisciplinary student community in the math department at Morgan. In the spirit of expanding this work, Dr. Hulse intends to start an unsolved problems seminar. This is partially meant to showcase the

work in this proposed research program to the student population at Morgan, but mainly to serve as a forum to present interesting unsolved problems in mathematics, introduce computational exploratory techniques, cultivate abstract problem-solving skills, and to provoke undergraduate and graduate research.

Dr. Hulse is experienced at supervising student research and independent study, having done so at Colby College as a visiting assistant professor and currently at Morgan, having overseen projects in analytic number theory, cryptography, and combinatorics. He intends to set aside smaller questions from this research program that can be tackled as projects for students. MSU offers a Senior Seminar course in which undergraduate math majors work on independent research projects under the guidance of faculty members. Students in this course are required to write a paper and present on their work at MSU's Undergraduate and Graduate Research Symposium. This course would serve as an excellent venue for serving interested undergraduate participants.

Additionally, Dr. Hulse plans to employ one graduate and two undergraduate students as part-time research assistants. Ideally, the undergraduate students would also be working with Dr. Hulse in the Senior Seminar and the graduate student would be Dr. Hulse's thesis advisee.

Undergraduate students would be chosen among those that have satisfactorily completed the calculus sequence at MSU or an equivalent and have demonstrated to the PI significant mathematical maturity. This would most likely be one of the 10-15 math majors in the department per year, or alternately students may be recruited from CS, engineering or physics. The positions will be advertised in the math club, senior seminar, the student chapter of American Statistical Association, in the weekly problem-solving sessions, and in-class announcements.

The graduate student assistant must have completed the graduate-level courses in abstract algebra and complex analysis or else be committed to completing those courses within a year. MSU currently has about eight masters students and twelve Ph.D students to recruit from.

Much of the exploratory work on this proposal was investigated using the Python-based, free and open-source computer algebra system, SageMath [3]. Dr. Hulse will train these students in using SageMath to aid in the research program by testing conjectures, compiling exploratory data and numerically verifying parts of proofs. In the process, the PI will use the experience to educate the students on different areas of analytic and computational number theory, fields that have close ties to industry sectors such as cryptography. To aid in this, the PI is requesting funds for computers and a professional subscription to CoCalc [1], the SageMath-powered cloud computing and project management service.

The more precise details of how the student research assistants will be trained and supervised throughout the course of this grant are outlined in the Student Mentoring Plan.

Dissemination of Research

Dr. Hulse and his collaborators are quite practiced now at producing regular publications in peer-reviewed journals, and would continue to regularly submit publications on new research throughout the life of this project. The PI also intends to present all research

findings at conferences, colloquia, invited talks, and workshops, and also intends to prepare his student research assistants to present their work and contributions at poster sessions at some of these events. To this end, funding for conference travel, both for Dr. Hulse and for his student assistants, is also requested and the specifics of this are outlined in the budget. Specifically, the PI is requesting funding to the annual Joint Mathematics Meetings and Maine-Québec Number Theory Conferences for the duration of this grant.

Evaluation of Success

An outline of the proposed research plan can be found in the section on Intellectual Merit. It should be noted that while this is the intended course of study, it is unlikely the research program will exactly follow this process, it may be necessary at times to backtrack and try variations of a different construction or follow unanticipated ideas along different routes. As new things are discovered the parameters of the program may change. Regular academic progress would be measured by regular production of publications, likely one or two a year, in peer-reviewed journals contributing to progress toward the main result over the course of the program, as well as presentations of these results at conferences.

The success of the educational goals of this program will be qualitatively assessed by the MSU Mathematics Department Chair, Dr. Asamoah Nkwanta, and quantitatively assessed by metrics such as increased number of math majors and minors, number of participants in the new seminar and the mathematical competition sessions, and the successful training of student research assistants who will in turn meaningfully contribute to the program in the form of presentations, posters and papers.

Intellectual Merit

The Object of Interest: A Sum of Squares

If we scale a rational right triangle by an integer, m, the result is still a rational right triangle but with area tm^2. So without loss of generality, we only need to be concerned with the case where t is square-free. Furthermore, a square-free integer, t, is a congruent number if and only if t is the square-free part of the area of a right triangle with integer-length sides that are relatively prime, a *primitive* right triangle. Henceforth we let $(a, b, c) \in \mathbb{N}^3$ denote an integral right triangle, where $a^2 + b^2 = c^2$, and let t denote a square-free integer.

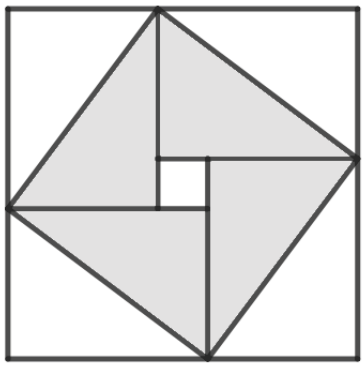

FIGURE 2. A tiling of rational right triangles and three rational squares.

There is a well-known [7] correspondence between pythagorean triangles and arithmetic progressions of three squares. This correspondence can, perhaps, be most easily visualized geometrically as a tiling of right-triangles forming a progression of concentric squares, as is

illustrated in Fig. 2. Algebraically, given all integral triangles (a, b, c) with square-free part of the area t, we have a one-to-one correspondence with integer pairs $(m, n) \in \mathbb{N}^2$ such that $m^2 - tn^2$ and $m^2 + tn^2$ are squares, given by:

$$\begin{array}{rcccl} (a,b,c) & \to & (c, \sqrt{2ab/t}) & = & (m,n) \\ (m,n) & \to & \left(\dfrac{\sqrt{m^2+tn^2}+\sqrt{m^2-tn^2}}{2}, \dfrac{\sqrt{m^2+tn^2}-\sqrt{m^2-tn^2}}{2}, m \right) & = & (a,b,c). \end{array} \tag{1}$$

Now let $\tau : \mathbb{Z} \to \{0, 1\}$ be the arithmetic function where $\tau(n) = 1$ if and only if n is a square, and let $X > 0$. We observe from the above correspondence that a square-free t is *not* a congruent number if and only if $s_t(X)$ is the constant-zero function where

$$s_t(X) := \sum_{m=1}^{X} \sum_{n=1}^{X} \tau(m^2 - tn^2)\tau(m^2 + tn^2). \tag{2}$$

Indeed, given that any m contributes to the sum if and only if m is the hypotenuse of a rational right triangle, and if m is such a hypotenuse then so is km for all $k \in \mathbb{N}$, it follows that

$$s_t(X) = \sum_{h_t \le X} \left\lfloor \frac{X}{h_t} \right\rfloor, \tag{3}$$

where the sum is over the hypotenuses, h_t, of primitive right triangles with square-free part of the area t, and $\lfloor x \rfloor$ denotes the greatest integer less than x. We note that there are infinitely many such h_t for any given t, but that the number of them is small relative to the size of $\log X$.

It will eventually be useful to consider a variant form of $s_t(X)$. We note that

$$s_t(X) := \sum_{m=1}^{X} \sum_{n=1}^{X} \tau(m^2 - tn^2)\tau(m^2 + tn^2) = \sum_{m=1}^{X^2} \sum_{n=1}^{X^2} \tau(m-n)\tau(m+n)\tau(m)\tau(tn) \tag{4}$$

and so we define $S_t(X)$ as

$$S_t(X) := \sum_{m=1}^{X} \sum_{n=1}^{X} \tau(m-n)\tau(m+n)\tau(m)\tau(tn) = \sum_{h_t \le X} \left\lfloor \frac{X^{\frac{1}{2}}}{h_t} \right\rfloor. \tag{5}$$

From here it is not difficult to prove using properties of the corresponding elliptic curve, $E_t : y^2 = x^3 - t^2 x$, that

$$S_t(X) = C_t X^{\frac{1}{2}} + O_t\left(\log X\right), \tag{6}$$

where C_t is given by the convergent infinite or empty sum,

$$C_t := \sum_{h_t} \frac{1}{h_t}. \tag{7}$$

Ultimately, the PI and his collaborators plan to obtain an asymptotic estimate of $S_t(X)$ from the meromorphic properties of shifted Dirichlet series of theta functions, and thus create a new independent criteria for the congruence of t.

Connection to Automorphic Forms: Theta Sums

Let $\mathbb{H} \subset \mathbb{C}$ denote the upper-half plane,

$$\mathbb{H} := \{x + iy \in \mathbb{C} \mid x \in \mathbb{R}, y > 0\}, \tag{8}$$

and let $z = x + iy \in \mathbb{H}$. For $N \in \mathbb{N}$, let $\Gamma_0(N)$ denote the congruence subgroup

$$\Gamma_0(N) := \left\{ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in SL_2(\mathbb{Z}) \;\middle|\; N|C \right\}, \tag{9}$$

which acts on $\mathbb{H}$ by Möbius transformations. That is, for $\gamma = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma_0(N)$, we say that

$$\gamma z := \frac{Az + B}{Cz + D}. \tag{10}$$

We define the automorphic theta function, $\theta(z)$ to be

$$\theta(z) := \sum_{k \in \mathbb{Z}} e^{2\pi i k^2 z} = \sum_{n=0}^{\infty} r_1(n) e^{2\pi i n z}, \tag{11}$$

which is uniformly convergent and thus holomorphic on compact subsets of $\mathbb{H}$. For $\gamma \in \Gamma_0(4)$, it is known [20] that

$$\theta(\gamma z) = \left(\frac{C}{D}\right) \epsilon_D^{-1} \sqrt{Cz + D}\, \theta(z), \tag{12}$$

where $\left(\frac{C}{D}\right)$ denotes Shimura's extension of the Jacobi symbol and $\epsilon_D = 1$ or i depending on if $D \equiv 1$ or $3 \pmod 4$, respectively. We say that $\theta(z)$ is a weight-$\frac{1}{2}$ holomorphic form of level 4.

The Fourier coefficients of $\theta(z)$ coincide almost identically with a multiple of the square-indicator function, $r_1(n) = 2\tau(n)$, except when $n = 0$. From this it is easy to see that

$$S_t(X) = \frac{1}{16} \sum_{m,n \leq X} r_1(m-n) r_1(m+n) r_1(m) r_1(tn). \tag{13}$$

Now let Q be a large prime number such that $(Q, t) = 1$ and let χ and ψ be Dirichlet characters with conductor Q. Consider the shifted partial sums,

$$S^-_{\chi,\psi}(X) := \sum_{1 \leq m_1, n_1 \leq X} r_1(m-n) r_1(m) \chi(m) \psi(n), \tag{14}$$

$$S^+_{\chi,\psi}(X; t) := \sum_{1 \leq m_2, n_2 \leq X} r_1(n+m) r_1(tn) \overline{\chi(m)\psi(n)}. \tag{15}$$

The orthogonality properties for Dirichlet characters then give us that

$$\sum_{\chi,\psi\in\widehat{(\mathbb{Z}/Q\mathbb{Z})^\times}} S^-_{\chi,\psi}(X)S^+_{\chi,\psi}(X;t) = 16S_t(X) + f(X,Q,t). \tag{16}$$

where $f(X,Q,t)$ becomes smaller as Q becomes large relative to X. Indeed, if $Q > X$ then $f(X,Q,t) = 0$.

Explicit asymptotic formulas for these shifted partial sums can be obtained by considering the analytic properties of the shifted multiple Dirichlet series,

$$D^-_{\chi,\psi}(s,w) := \sum_{m,n\geq 1} \frac{r_1(m-n)r_1(m)\chi(m)\psi(n)}{m^s n^w}, \tag{17}$$

$$D^+_{\chi,\psi}(s,w;t) := \sum_{m,n\geq 1} \frac{r_1(t(m-n))r_1(m)\overline{\psi(m-n)\chi(n)}}{m^s n^w}. \tag{18}$$

Objects such as these are well-studied by Dr. Hulse and his collaborators, generally in the case of shifted sums of automorphic forms of L-functions, [11, 10, 14, 15, 16], and more recently and specifically in their investigation of the Gauss Circle Problem [13, 12]. Indeed, using spectral methods, we can obtain meromorphic continuations of these objects, and from these effectively estimate the inverse Mellin Transforms.

It is worth noting that (16) is not the only possible way one might choose to decompose $S_t(X)$ into more manageable sums, and indeed other variants of the requisite Dirichlet series may be more tractable. This example mainly serves as a proof of concept.

Spectral Expansions of Shifted Sums

The term "shifted convolution sum" generally refers to an object resembling

$$\sum_{m=1}^{\infty} \lambda_1(m+h)\lambda_2(m)W(m,h), \tag{19}$$

where $W(m,h)$ is a weight function that, ideally, has manageable coupling between m and h and decays reasonably over specified ranges. These sums are sometimes referred to as "sums of the additive divisor problem type" [17], as the case when the λ_is are additive divisor functions has been long studied in classical analytic number theory. They were notably used by Atkinson [5] to compute asymptotics of the fourth moment of the Riemann zeta function. Other sums of this form, where the λ_is arose from automorphic forms, were first constructed by Selberg [19] where they resembled

$$\sum_{n=1}^{\infty} \frac{a(n)\overline{b(n+h)}}{(2n+h)^s}, \tag{20}$$

when $\mathfrak{Re}\, s > 1$ and $a(n)$ and $b(n)$ are the Fourier coefficients of holomorphic cusp forms f and g. Selberg studied these by replacing the real-analytic Eisenstein series in the Rankin-Selberg convolution with a Poincaré series and then untiling as in the convolution. Similarly, by making use of another well-studied Poincaré series,

$$P_h(z,s) := \sum_{\gamma \in \Gamma_\infty \backslash \Gamma_0(N)} (\mathfrak{Im}\, \gamma z)^s e^{2\pi i h \gamma z}, \tag{21}$$

for $h \geq 1$, which is square integrable for sufficiently large $\mathfrak{Re}\, s$, we can expand the Petersson inner product $\langle (\mathfrak{Im}\, *)^k f\overline{g}, P_h(*,\overline{s})\rangle$ either by untiling as in the Rankin-Selberg convolution or by taking the spectral expansion of $P_h(z,s)$. In the case where f and g are holomorphic cusp forms for $SL_2(\mathbb{Z})$ of even weight $k > 0$, the equivalence of these two expansions gives that

$$\begin{aligned} D_{f,g}(s;h) &:= \sum_m \frac{a(m-h)\overline{b(m)}}{m^{s+k-1}} \qquad (22)\\ &= \frac{(4\pi)^k h^{\frac{1}{2}-s}}{\Gamma(s+k-1)\Gamma(s)} \left(\sum_j \overline{\lambda_j(h)\rho_j(1)}\Gamma(s-\tfrac{1}{2}+it_j)\Gamma(s-\tfrac{1}{2}-it_j)\overline{\langle y^k \overline{f} g, u_j\rangle} \right.\\ &\quad \left. + \frac{1}{2\pi i}\int_{(0)} \frac{\sigma_{2z}(h) h^{-z}\Gamma(s-\frac{1}{2}+z)\Gamma(s-\frac{1}{2}-z)}{2\zeta^*(1-2z)\zeta^*(1+2z)} \overline{\langle y^k \overline{f} g, E^*(*,\tfrac{1}{2}+z)\rangle}\, dz \right), \end{aligned}$$

when $\mathfrak{Re}\, s > 1$. The discrete part of the spectrum arises from a linear combination of u_js, weight-zero Mass forms, eigenfunctions of the Laplacian for $SL_2(\mathbb{Z})\backslash\mathbb{H}$, with Fourier-Whittaker expansion

$$u_j(z) = \sum_{|m|\neq 0} \rho_j(\tfrac{m}{|m|})\lambda_j(|m|)\sqrt{y} K_{it_j}(2\pi|m|y) e^{2\pi i m x} \tag{23}$$

and eigenvalue $\frac{1}{4}+t_j^2$ which are orthonormal with respect to the Petersson inner product. The continuous part of the spectrum is due to $E^*(z,s)$, the completed real-analytic Eisenstein series for $SL_2(\mathbb{Z})$. Known growth properties of $\langle y^k \overline{f} g, u_j\rangle$ due to Watson [22] tells us that the discrete sum is locally uniformly convergent for all s except at poles, and known properties of the Rankin-Selberg convolution, $\langle y^k \overline{f} g, E^*(*,\frac{1}{2}+z)\rangle$, similarly gives convergence of the continuous part of the spectrum to all non-polar s, though we obtain residual terms as we move past the lines $\mathfrak{Re}\, s = \frac{1}{2} - r$ for $r \in \mathbb{Z}_{\geq 0}$.

From this, it is not hard to use the above formula to produce a meromorphic continuation of $D_{f,g}(s;h)$ to all $s \in \mathbb{C}$ and also thus for the shifted multiple Dirichlet series,

$$Z_{f,g}(s,w) := \sum_{m,h=1}^{\infty} \frac{a(m-h)b(m)}{m^{s+k-1}h^w} = \sum_{h=1}^{\infty} \frac{D_{f,g}(s;h)}{h^w} \tag{24}$$

$$= \frac{(4\pi)^k}{\Gamma(s+k-1)\Gamma(s)} \left(\sum_j L(s+w-\tfrac{1}{2}, \overline{u_j}) \overline{\rho_j(1)} \Gamma(s-\tfrac{1}{2}+it_j)\Gamma(s-\tfrac{1}{2}-it_j) \overline{\langle y^k \overline{f} g, u_j \rangle} \right.$$

$$\left. + \frac{1}{2\pi i} \int_{(\frac{1}{2})} \frac{\zeta(s+w+z-1)\zeta(s+w-z)\Gamma(s+z-1)\Gamma(s-z)}{2\zeta^*(2-2z)\zeta^*(2z)} \overline{\langle y^k \overline{f} g, E^*(*,z) \rangle}\, dz \right),$$

to all $(s,w) \in \mathbb{C}^2$. Here,

$$L(s, \overline{u_j}) := \sum_{m=1}^{\infty} \frac{\overline{\lambda(m)}}{m^s} \tag{25}$$

which is locally uniformly convergent for $\Re s > 1$ and has a well-understood analytic continuation to all $s \in \mathbb{C}$. Similar constructions also exist for modular forms for $\Gamma_0(N)$ and of odd-integer or half-integer weight, though analysis of the continuous part of the spectrum needs to account for inequivalent cusps that make explicit computations of the residues at poles significantly more complicated.

We would like to consider a variant of this construction where f and g are replaced with theta functions. First, for square-free t and Dirichlet character χ with prime conductor Q with $(t,Q)=1$, define the twisted theta function

$$\theta_{t,\chi}(z) = \sum_{m=0}^{\infty} r_1(m)\chi(m) e^{2\pi i m t z} = \overline{\chi(t)} \sum_{m=0}^{\infty} r_1(tm)\chi(m) e^{2\pi i m z} \tag{26}$$

and that, for $\gamma = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma_0(4tQ^2)$, $\theta_{t,\chi}(z)$ satisfies

$$\theta_{t,\chi}(\gamma z) = \chi^2(D) \left(\tfrac{t}{D}\right) \left(\tfrac{C}{D}\right) \epsilon_D^{-1} \sqrt{Cz+D}\, \theta_{t,\chi}(z). \tag{27}$$

So we say $\theta_{t,\chi}(z)$ is a weight-$\frac{1}{2}$ holomorphic form of level $4tQ^2$ and character $\chi^2\left(\frac{t}{\cdot}\right)$. For simplicity of notation we let $\theta_\chi(z) := \theta_{1,\chi}(z)$ and let $\left(\frac{1}{\cdot}\right) := 1$.

With this in mind, we are tempted to proceed as in the case of holomorphic cusp forms, f and g, and consider the sums

$$\sum_{n=1}^{\infty} \frac{\psi(n) \langle (\mathfrak{Im}\, *)^{\frac{1}{2}} \theta_\chi \overline{\theta}, P_n(*,\overline{s}) \rangle}{n^{w-s+\frac{1}{2}}} \tag{28}$$

and

$$\sum_{n=1}^{\infty} \frac{\overline{\chi(n)} \psi(t) \langle (\mathfrak{Im}\, *)^{\frac{1}{2}} \theta_{t,\overline{\psi}} \overline{\theta}, P_n(*,\overline{s}) \rangle}{n^{w-s+\frac{1}{2}}} \tag{29}$$

with the expectation that they will give us spectral expansions of $D^-_{\chi,\psi}(s,w)$ and $D^+_{\chi,\psi}(s,w;t)$, respectively, but the reality is more complicated. Unfortunately, θ, θ_χ, and $\theta_{t,\chi}$ are often not cusp forms and so lack the rapid decay that is necessary for the inner-products $\langle (\mathfrak{Im}\, *)^{\frac{1}{2}}\theta_\chi\overline{\theta}, P_n(*,\overline{s})\rangle$ and $\langle (\mathfrak{Im}\, *)^{\frac{1}{2}}\theta_{t,\overline{\psi}}\overline{\theta}, P_n(*,\overline{s})\rangle$ to be well-defined and to obtain a spectral expansion like in (22) and (24).

To accommodate this, we can modify $\theta_\chi\overline{\theta}$ and $\theta_{t,\overline{\psi}}\overline{\theta}$ in the above constructions by subtracting linear combinations of special values of Eisenstein series, or other components of Eisenstein series, so that we have objects that vanish at the cusps of the congruence subgroups and allow us to expand the inner products as we would like. This is not theoretically challenging, indeed similar constructions were achieved by the PI and his collaborators in [13, 12] and is spiritually similar to the regularization techniques of Zagier [23] but, practically, it is an excessively meticulous undertaking. The bookkeeping required to catalogue the behavior of $\theta_\chi\overline{\theta}$ and $\theta_{t,\overline{\psi}}\overline{\theta}$ at each of the cusps, which depends on each of the parameters t, Q, χ, ψ, is extensive. This is also not a trivial concern, as it is likely that these subtracted Eisenstein series will contribute the rightmost poles of $D^-_{\chi,\psi}(s,w)$ and $D^+_{\chi,\psi}(s,w;t)$, and so complete understanding of this will be instrumental in computing C_t. This is also likely to be an avenue for computational exploration and thus potential undergraduate research.

Once the analytic properties of $D^-_{\chi,\psi}(s,w)$ and $D^+_{\chi,\psi}(s,w;t)$ are cataloged, Dr. Hulse and his collaborators will use inverse Mellin transforms to translate this information into asymptotic estimates of $S^-_{\chi,\psi}(X)$ of $S^+_{\chi,\psi}(X;t)$. This approach is a standard one in analytic number theory, though it becomes increasingly difficult as the analytic objects become more sophisticated and thus balancing terms requires a fair amount of patience and ingenuity. A sample of some of the techniques familiar to Dr. Hulse and his collaborators can be found in Section 6 of [16].

Relevant Expertise

Every component of this research program is a direct analog of a computation or construction made by Dr. Hulse and his collaborators on previous projects in published and submitted works, [11, 10, 14, 15, 16, 13, 12], as noted above. The proposed object of study that is relevant to the congruent number problem, $S_t(X)$, is a shifted sum and Dr. Hulse and his collaborators are experts on the subjects of shifted convolution sums of automorphic forms, spectral decompositions, and sharp cutoff estimates from inverse Mellin transforms.

Challenges and Contribution to the Field

A very broad outline of the steps that need to be followed for the successful completion of this project would be:

1) Classify all growth behavior at the cusps of the products of the variant theta functions in different combinations. This information can then be used to subtract the correct linear combination of Eisenstein series to create a square-integrable object.

2) Take the inner product of the now square-integrable objects against the relevant Poincaré series and use this to compute the spectral expansion of a shifted multiple Dirichlet series. Then catalog the residues at relevant poles and discern growth properties of the function on vertical strips. Perhaps the greatest difficulty will probably be calculating the cancellation of the different contributions to the continuous part of the spectrum and discerning the rightmost pole.

3) Use the information about the growth of the function and its residues to compute an inverse Mellin transform of the shifted Dirichlet series and balance terms to get the asymptotics of the relevant shifted partial sum.

4) Average the product of the shifted partial sums over the characters, balancing the character with the size of the partial sum, and derive a main term.

5) If the main term is effectively computable, this would provide a solution to the Congruent Number Problem.

It is yet unclear how the analytic properties of the spectral decomposition of shifted Dirichlet series might conspire to produce an alternate formulation of C_t, as given in (7), and whether there will be an explicit formula. This is partly why it is a fruitful target for further study. It is plausible that computing C_t will somehow provide independent confirmation of Tunnell's criterion, which would prove it to be a solution to the Congruent Number Problem and so would provide a a special case of the Birch and Swinnerton-Dyer Conjecture in the case of the Elliptic Curve family $E : y^2 = x^3 - t^2x$. This could be groundbreaking to the field of analytic number theory.

There are many anticipated challenges afforded by the careful investigation of the continuous contributions of the spectral expansion, which likely yield significant and non-trivial cancellation. This work will require extreme patience and attention to detail. There are also speculative or unforeseen challenges. It may be the case that the spectral computations for C_t might simply depend explicitly on the existence on the rational points of E and so only reveal what is known but in a surprising way. Alternately, it may be extremely difficult to prove that the constant C_t is ever nonzero from the spectral contributions. However, even if the ultimate results are modest, this construction would demonstrate a new relationship between a family of elliptic curves and the spectral expansion of shifted multiple Dirichlet series, an undiscovered connection between analytic and algebraic objects which could have far-reaching and unforeseen consequences.

Broader Impact

As described in more detail in the earlier section on student participation, Dr. Hulse is continuing his work to grow an enthusiastic and engaged student community in the math department at MSU, an Historically Black College and University. Engaging the student community at Morgan and preparing them for successful, math-intensive careers helps to elevate the participation of underrepresented groups in STEM fields in industry, both generally but also in the specific areas of computational number theory, which have relevant

ties to industries like those invested in cryptography. This is relevant as Cybersecurity interests, through private firms and the government by way of the Department of Defense and specifically the National Security Agency, provide significant employment opportunities in the greater Baltimore-D.C. metropolitan area, while also serving the welfare of the United States.

Furthermore, significant progress in a project such as this one would contribute to the research profile of the math department at Morgan, Maryland's preeminent public urban research university. Morgan is a beacon for academic achievement in the Baltimore community, and notable successes can serve to capture the public imagination and foster general enthusiasm for mathematics.

MORGAN STATE UNIVERSITY
GROWING THE FUTURE • LEADING THE WORLD

School of Computer, Mathematical and Natural Sciences
Office of the Dean

February 23, 2018

To whom it may concern:

I strongly endorse the grant proposal entitled, "Shifted Convolution Sums for The Congruent Number Problem," being submitted by Assistant Prof. Thomas A. Hulse under the Excellence in Research (EiR) category of the NSF Call HBCU- 18-522. This Number Theory related proposal aims to advance our understanding of the Congruent Number Problem, which asks for a description of all numbers that are the areas of right triangles with rational-length sides. Dr. Hulse has discovered that this problem is closely related to the asymptotic behaviour of new families of shifted convolution sums of Fourier coefficients of theta functions. While the Congruent Number Problem is over a thousand years old, Dr. Hulse's approach represents the cutting edge in Number Theoretic Research.

The most important aspect of the proposed research is the training of undergraduate students in advanced Number Theoretic techniques and famous unsolved problems in mathematics, including the Birch-Swinnerton-Dyer Conjecture, a Millennium Prize Problem. The proposed project also has important applications in the area of computational number theory, and so by fostering student research on this subject will thus have an impact on industry and government in fields such as cryptography. Dr. Hulse has made substantial steps toward a promising research career and has been active in recruiting and mentoring undergraduate and graduate students at Morgan State. This funding will position him boost our recruitment and training of underrepresented minorities in advanced STEM research.

As part of my commitment, Dr. Hulse will have the full support of the Dean's Office staff for administrative assistance for grant management and paperwork, support for recruitment of students, and dissemination of research results. To this cause, I am hiring a grant management specialist to assist the faculty like him who are awarded grants. In addition, I will ensure that he has the needed release time to implement, supervise, and oversee the progress of the project.

Sincerely,

Hongtao Yu, Ph.D.

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MORGAN STATE UNIVERSITY

GROWING THE FUTURE • LEADING THE WORLD

Department of Mathematics

February 21, 2018

National Science Foundation
2415 Eisenhower Avenue
Alexandria, VA 22314

Re: NSF 18-522 HBCU-UP Excellence in Research Projects

Title: Excellence in Research: Shifted Convolution Sums for The Congruent Number Problem

Dear Review Panel:

I am pleased to write this letter of support for Dr. Thomas Hulse's NSF proposal: Shifted Convolution Sums for The Congruent Number Problem. I strongly support his proposal.

Dr. Hulse is an enthusiastic faculty member and researcher with a unique commitment for working with mathematics majors. His research area of interest is analytic number theory. My understanding is that this area of number theory involves using L-functions to study classical automorphic forms and related Diophantine equations. Along with his collaborators, he proposes to provide more insight into the classical congruent number problem by investigating the problem in the context of shifted Multiple Dirichlet series.

If the proposal submitted by Dr. Hulse is selected for funding by NSF, it is my intent to commit resources and/or provide services as detailed in the Project Description, or the Facilities, Equipment, and Other Resource sections of his proposal. In addition, I will make myself available to him for professional development and guidance on his proposal research goals and student mentoring plans.

Sincerely,

Asamoah Nkwanta

Asamoah Nkwanta, Ph.D.
Chair and Professor of Mathematics

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Thomas A. Hulse
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School of Computer, Mathematical and Natural Sciences
Morgan State University, 1700 E. Cold Spring Lane, Baltimore, MD 21251.
443-885-1009 (p); 443-885-8216 (f); Email: thomas.hulse@morgan.edu

(a) Professional Preparation

Colby College, Waterville, ME, USA	Mathematics and Physics	B.A. 2007
Brown University, Providence, RI, USA	Mathematics	Sc.M. 2009
Brown University, Providence, RI, USA	Mathematics	Ph.D. 2013
Queen's University, Kingston, ON, Canada	Analytic Number Theory	Postdoc 2013-2015

(b) Appointments

2017 – Present	Assistant Professor of Mathematics, Morgan State University
2015 – 2017	Visiting Assistant Professor of Mathematics, Colby College

(c) Products

Five Products Most Related to the Proposed Project

1. T. A. Hulse, D. Lowry-Duda, C. I. Kuan, A. Walker. Sign changes of coefficients and sums of coefficients of L-functions. *Journal of Number Theory*, Vol. 177, August **2017**, Pages 112-135. https://doi.org/10.1016/j.jnt.2017.01.007

2. T. A. Hulse, D. Lowry-Duda, C. I. Kuan, A. Walker. The Second Moment of Sums of Coefficients of Cusp Forms. *Journal of Number Theory*, Vol. 173, April **2017**, Pages 304-331. https://doi.org/10.1016/j.jnt.2016.09.005

3. T. A. Hulse, D. Lowry-Duda, C. I. Kuan, A. Walker. Short-Interval Averages of Sums of Fourier Coefficients of Cusp Forms. *Journal of Number Theory,* Vol. 173, April **2017**, Pages 394-415. DOI: https://doi.org/10.1016/j.jnt.2016.09.004

4. T. A. Hulse, E. Mehmet Kıral, C. I. Kuan, and Li-Mei Lim. Counting Square Discriminants. *Journal of Number Theory*, Vol. 162, May **2016**, Pages 255-274. https://doi.org/10.1016/j.jnt.2015.10.015

5. T. A. Hulse, D. Lowry-Duda, C. I. Kuan, A. Walker. The Laplace Transform of the Second Moment in the Gauss Circle Problem. Accepted Pending Revision to *Algebra & Number Theory.* Submitted 2017. https://arxiv.org/abs/1705.04771

Five Other Significant Products

6. J. Hoffstein and T. A. Hulse. Multiple Dirichlet Series and Shifted Convolutions. *Journal of Number Theory*, Vol. 161, April **2016**, Pages 457-533. https://doi.org/10.1016/j.jnt.2015.10.001

7. T. A. Hulse, E. Mehmet Kıral, C. I. Kuan, and Li-Mei Lim. The Sign of Fourier Coefficients of Half-Integral Weight Cusp Forms. *International Journal of Number Theory*, Vol. 8, No. 3, **2012,** Pages 749-762. https://doi.org/10.1142/S179304211250042X

8. T. A. Hulse, M. R. Murty. Bertrand's postulate for number fields. *Colloquium Mathematicum,* Vol. 147, No. 2, **2017**, 165-180. https://doi.org/10.4064/cm7048-9-2016

9. T. A. Hulse, D. Lowry-Duda, C. I. Kuan, A. Walker. Second Moments in the Generalized Gauss Circle Problem. Submitted for Publication. 2017. https://arxiv.org/abs/1703.10347

10. T. A. Hulse. Triple Shifted Sums of Automorphic L-functions. Ph.D. Thesis. Brown University. 2013. https://doi.org/10.7301/Z0RB72ZC

(d) Synergistic Activities

- Journal Referee: 1) *Journal of Number Theory,* 2016-Present; 2) *Publicationes Mathematicae Debrecen,* 2017; 3) *Involve, a Journal of Mathematics,* 2017.

HRD PROPOSAL BUDGET

YEAR 1

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
Morgan State University

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Thomas A Hulse

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUM	Funds Requested From NSF	Non-Federal Matching Funds	Total Project Cost
1. **Thomas A Hulse - Assistant Professor**	0.00	0.00	2.00	$ **14,211**	$ **0**	$ **14,211**
2.						
3.						
4.						
5.						
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	**0**	**0**
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	**14,211**	**0**	**14,211**
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	**0**	**0**
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	**0**	**0**
3. (**0**) GRADUATE STUDENTS				**0**	**0**	**0**
4. (**0**) UNDERGRADUATE STUDENTS				**0**	**0**	**0**
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	**0**	**0**
6. (**0**) OTHER				**0**	**0**	**0**
TOTAL SALARIES AND WAGES (A + B)				**14,211**	**0**	**14,211**
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**1,279**	**0**	**1,279**
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**15,490**	**0**	**15,490**
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				**0**	**0**	**0**
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**2,010**	**0**	**2,010**
2. INTERNATIONAL				**1,091**	**0**	**1,091**
F. PARTICIPANT SUPPORT COSTS						
1. STIPENDS $ **28,000**						
2. TRAVEL **3,000**						
3. SUBSISTENCE **0**						
4. OTHER **0**						
(**3**) TOTAL PARTICIPANT COSTS				**31,000**	**0**	**31,000**
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				**8,300**	**0**	**8,300**
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**0**	**0**	**0**
3. CONSULTANT SERVICES				**0**	**0**	**0**
4. COMPUTER SERVICES				**999**	**0**	**999**
5. SUBAWARDS				**0**	**0**	**0**
6. OTHER				**0**	**0**	**0**
TOTAL OTHER DIRECT COSTS				**9,299**	**0**	**9,299**
H. TOTAL DIRECT COSTS (A THROUGH G)				**58,890**	**0**	**58,890**
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 51.0000, Base: 27890)**						
TOTAL INDIRECT COSTS (F&A)				**14,224**	**0**	**14,224**
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**73,114**	**0**	**73,114**
K. SMALL BUSINESS FEE				**0**	**0**	**0**
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ **73,114**	$ **0**	$ **73,114**

M. COST SHARING PROPOSED LEVEL $ **0** | AGREED LEVEL IF DIFFERENT $

PI/PD NAME
Thomas A Hulse

ORG. REP. NAME*

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

HRD PROPOSAL BUDGET

YEAR 2

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
Morgan State University

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Thomas A Hulse

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUM	Funds Requested From NSF	Non-Federal Matching Funds	Total Project Cost
1. **Thomas A Hulse - Assistant Professor**	0.00	0.00	2.00	$ **14,637**	$ **0**	$ **14,637**
2.						
3.						
4.						
5.						
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	**0**	**0**
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	**14,637**	**0**	**14,637**
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	**0**	**0**
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	**0**	**0**
3. (**0**) GRADUATE STUDENTS				**0**	**0**	**0**
4. (**0**) UNDERGRADUATE STUDENTS				**0**	**0**	**0**
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	**0**	**0**
6. (**0**) OTHER				**0**	**0**	**0**
TOTAL SALARIES AND WAGES (A + B)				**14,637**	**0**	**14,637**
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**1,317**	**0**	**1,317**
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**15,954**	**0**	**15,954**
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				**0**	**0**	**0**
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**4,139**	**0**	**4,139**
2. INTERNATIONAL				**0**	**0**	**0**
F. PARTICIPANT SUPPORT COSTS						
1. STIPENDS $ **28,000**						
2. TRAVEL **3,000**						
3. SUBSISTENCE **0**						
4. OTHER **0**						
(**3**) TOTAL PARTICIPANT COSTS				**31,000**	**0**	**31,000**
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				**3,300**	**0**	**3,300**
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**0**	**0**	**0**
3. CONSULTANT SERVICES				**0**	**0**	**0**
4. COMPUTER SERVICES				**999**	**0**	**999**
5. SUBAWARDS				**0**	**0**	**0**
6. OTHER				**0**	**0**	**0**
TOTAL OTHER DIRECT COSTS				**4,299**	**0**	**4,299**
H. TOTAL DIRECT COSTS (A THROUGH G)				**55,392**	**0**	**55,392**
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 51.0000, Base: 24392)**						
TOTAL INDIRECT COSTS (F&A)				**12,440**	**0**	**12,440**
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**67,832**	**0**	**67,832**
K. SMALL BUSINESS FEE				**0**	**0**	**0**
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ **67,832**	$ **0**	$ **67,832**

M. COST SHARING PROPOSED LEVEL $ **0** AGREED LEVEL IF DIFFERENT $

PI/PD NAME
Thomas A Hulse

ORG. REP. NAME*

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INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

HRD PROPOSAL BUDGET

YEAR 3

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
Morgan State University

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Thomas A Hulse

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUM	Funds Requested From NSF	Non-Federal Matching Funds	Total Project Cost
1. **Thomas A Hulse - Assistant Professor**	0.00	0.00	2.00	$ **15,076**	$ **0**	$ **15,076**
2.						
3.						
4.						
5.						
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	**0**	**0**
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	2.00	**15,076**	**0**	**15,076**
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	**0**	**0**
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	**0**	**0**
3. (**0**) GRADUATE STUDENTS				**0**	**0**	**0**
4. (**0**) UNDERGRADUATE STUDENTS				**0**	**0**	**0**
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	**0**	**0**
6. (**0**) OTHER				**0**	**0**	**0**
TOTAL SALARIES AND WAGES (A + B)				**15,076**	**0**	**15,076**
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**1,357**	**0**	**1,357**
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**16,433**	**0**	**16,433**
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				**0**	**0**	**0**
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**2,372**	**0**	**2,372**
2. INTERNATIONAL				**1,091**	**0**	**1,091**
F. PARTICIPANT SUPPORT COSTS						
1. STIPENDS $ **28,000**						
2. TRAVEL **3,000**						
3. SUBSISTENCE **0**						
4. OTHER **0**						
(**3**) TOTAL PARTICIPANT COSTS				**31,000**	**0**	**31,000**
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				**3,300**	**0**	**3,300**
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**0**	**0**	**0**
3. CONSULTANT SERVICES				**0**	**0**	**0**
4. COMPUTER SERVICES				**999**	**0**	**999**
5. SUBAWARDS				**0**	**0**	**0**
6. OTHER				**0**	**0**	**0**
TOTAL OTHER DIRECT COSTS				**4,299**	**0**	**4,299**
H. TOTAL DIRECT COSTS (A THROUGH G)				**55,195**	**0**	**55,195**
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) **MTDC (Rate: 51.0000, Base: 24195)**						
TOTAL INDIRECT COSTS (F&A)				**12,339**	**0**	**12,339**
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**67,534**	**0**	**67,534**
K. SMALL BUSINESS FEE				**0**	**0**	**0**
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ **67,534**	$ **0**	$ **67,534**

M. COST SHARING PROPOSED LEVEL $ **0** AGREED LEVEL IF DIFFERENT $

PI/PD NAME
Thomas A Hulse

ORG. REP. NAME*

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INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

*ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

HRD
PROPOSAL BUDGET

Cumulative

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (months) Proposed	Granted
AWARD NO.		

ORGANIZATION
Morgan State University

PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR
Thomas A Hulse

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUM	Funds Requested From NSF	Non-Federal Matching Funds	Total Project Cost
1. **Thomas A Hulse - Assistant Professor**	0.00	0.00	6.00	$ **43,924**	$ **0**	$ **43,924**
2.						
3.						
4.						
5.						
6. (**0**) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	**0**	**0**	**0**
7. (**1**) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	6.00	**43,924**	**0**	**43,924**
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)						
1. (**0**) POST DOCTORAL SCHOLARS	0.00	0.00	0.00	**0**	**0**	**0**
2. (**0**) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	**0**	**0**	**0**
3. (**0**) GRADUATE STUDENTS				**0**	**0**	**0**
4. (**0**) UNDERGRADUATE STUDENTS				**0**	**0**	**0**
5. (**0**) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				**0**	**0**	**0**
6. (**0**) OTHER				**0**	**0**	**0**
TOTAL SALARIES AND WAGES (A + B)				**43,924**	**0**	**43,924**
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				**3,953**	**0**	**3,953**
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				**47,877**	**0**	**47,877**
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.)						
TOTAL EQUIPMENT				**0**	**0**	**0**
E. TRAVEL 1. DOMESTIC (INCL. U.S. POSSESSIONS)				**8,521**	**0**	**8,521**
2. INTERNATIONAL				**2,182**	**0**	**2,182**
F. PARTICIPANT SUPPORT COSTS						
1. STIPENDS $ **84,000**						
2. TRAVEL **9,000**						
3. SUBSISTENCE **0**						
4. OTHER **0**						
(**9**) TOTAL PARTICIPANT COSTS				**93,000**	**0**	**93,000**
G. OTHER DIRECT COSTS						
1. MATERIALS AND SUPPLIES				**14,900**	**0**	**14,900**
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				**0**	**0**	**0**
3. CONSULTANT SERVICES				**0**	**0**	**0**
4. COMPUTER SERVICES				**2,997**	**0**	**2,997**
5. SUBAWARDS				**0**	**0**	**0**
6. OTHER				**0**	**0**	**0**
TOTAL OTHER DIRECT COSTS				**17,897**	**0**	**17,897**
H. TOTAL DIRECT COSTS (A THROUGH G)				**169,477**	**0**	**169,477**
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)						
TOTAL INDIRECT COSTS (F&A)				**39,003**	**0**	**39,003**
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				**208,480**	**0**	**208,480**
K. SMALL BUSINESS FEE				**0**	**0**	**0**
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ **208,480**	$ **0**	$ **208,480**

M. COST SHARING PROPOSED LEVEL $ **0** AGREED LEVEL IF DIFFERENT $

PI/PD NAME
Thomas A Hulse

ORG. REP. NAME*

FOR NSF USE ONLY		
INDIRECT COST RATE VERIFICATION		
Date Checked	Date Of Rate Sheet	Initials - ORG

***ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET**

Budget Justification

Excellence in Research: Shifted Convolution Sums for The Congruent Number Problem

A. Senior Project Personnel Salaries and Wages: Dr. Thomas A. Hulse

Year 1: $14,211
Year 2: $14,637
Year 3: $15,076

Dr. Thomas A. Hulse will serve as PI on this project. An Assistant Professor of the Department of Mathematics at Morgan State University (MSU), Dr. Hulse is noted for his work in the field of analytic number theory, having co-authored several papers for peer-reviewed journals concerning classical automorphic forms and L-functions, further disseminating this research at invited talks and conferences. In particular, he is an expert on the application of shifted sums of Fourier coefficients of automorphic forms to problems in number theory, which is one of the major research goals of this project. He is also experienced at supervising student research and independent study in the field of number theory, both at MSU (an HBCU) and elsewhere, and so will use this project as an avenue for advancing underrepresented students in the STEM fields, another major goal of this project.

Dr. Hulse is committed for 2 summer months each year of the project. A rise of 3% per year for Dr. Hulse's current annual 9.5-month salary of $67,500 has been calculated for the period.

B. Other Personnel: None

C. Fringe Benefits

Year 1: $1,279
Year 2: $1,317
Year 3: $1,357

The PI, a fully appointed faculty member, receives two months summer funding per year, $14,21 for the first year, escalating by 3% in subsequent years. At the applicable 9% fringe rate for summer months.

D. Equipment: None.

E. Travel:

Year 1: $3,101
Year 2: $4,139
Year 3: $3,463

Maine-Québec - The Maine-Québec Number Theory Conference is an annual conference, usually in October, of many prominent number theorists that is held alternately at the University of Maine in Orono, Maine and Université Laval in Québec City, Canada. It would be an excellent event for the PI to present new findings to the North American number theory community. This is alternately listed under domestic and foreign travel, depending on the year.

Joint Meetings - The Joint Mathematics Meetings of the American Mathematical Society and the

Mathematical Association of America is largest math conference in the world and an ideal place for the PI to present findings to a wide audience, as well as student researchers to present their work at talks and poster sessions. In addition to travel expenses, there will be registration fees for AMS members. This is consistently listed under domestic travel.

Required Meeting Travel - All proposals are instructed to budget for the PI to attend a one to two day grantee meeting in the Washington, DC area every year of the project. Since the PI lives in Baltimore, this is just the cost of mileage and meals.

Unspecified Conference Travel - Not all conferences are foreseen by this budget over the span of the coming three years, and the PI may also find it necessary to accompany student research assistants to additional conferences and workshops that primarily serve their career ambitions. As such, speculative funding for additional conference travel is included.

a) Domestic

Year 1: $2,010
Year 2: $4,139
Year 3: $2,372

Year 1:
In 2018, the Maine-Québec Conference is in Québec City and so is listed under foreign travel.

In 2019 the Joint Meetings will be held in Baltimore, MD which is where the PI and Morgan State University are located. The only travel expense will be the registration fee.
Estimated Total Costs: $329

Required Meeting Travel to Washington, DC:
Estimated Total Costs: $181

- Mileage (80 miles a day for 2 days): $87
- Meals for 2 days ($47 per day): $94

Unspecified Conference Travel: $1500

Year 2:
In 2019 the Maine-Québec Conference will be in Orono, Maine. The estimated expenses are:
Estimated Total Costs: $791

- Air Travel: $300
- Transportation to and from airport: $50
- Hotel Accommodations for 2 nights: $300
- Meals for 3 days ($47 per day): $141

In 2020 the Joint Meetings will be held in Denver, CO. The estimated expenses are:
Estimated Total Costs: $1,667

- Air Travel: $500
- Transportation to and from airport: $50
- Hotel Accommodations for 4 nights: $600
- Meals for 4 days ($47 per day) : $188
- Registration Fee: $329

Required Meeting Travel to Washington, DC:
Estimated Total Costs: $181
Same as Year 1

Unspecified Conference Travel: $1500

Year 3:
In 2020, the Maine-Québec Conference is in Québec City and so is listed under foreign travel.

In 2021, the Joint Meetings will be in Washington, DC. Since this is manageable driving distance from Baltimore, MD, where the PI and Morgan State are located, cost of mileage and meals will be sufficient for travel.
Estimated Total Costs: $691

- Mileage (80 miles a day for 4 days): $174
- Meals for 4 days: $188
- Registration Fee: $329

Required Meeting Travel to Washington, DC:
Estimated Total Costs: $181
Same as Year 1

Unspecified Conference Travel: $1500

b) Foreign

Year 1: $1,091
Year 2: $0
Year 3: $1,091

All Foreign Travel is to Québec City, Canada, probably in October 2018 and October 2020, specific dates are not yet specified.

Year 1
In 2018 the Maine-Québec Conference conference will be in Québec City, Canada. The conference has yet to be scheduled but it likely to be in October of 2018. The estimated expenses are:
Estimated Total Costs: $1,091

- Air Travel: $600
- Transportation to and from airport: $50
- Hotel Accommodations for 2 nights: $300
- Meals for 3 days: $141

Year 2
There is no foreign travel planned for this year.

Year 3
In 2020, the Maine-Québec Conference conference will be at Québec City, Canada. The plan for this year is the same as Year 1.

F. Participant Support Costs:

Year 1: $31,000
Year 2: $31,000
Year 3: $31,000

a) Stipends:

Year 1: $28,000
Year 2: $28,000
Year 3: $28,000

One of the main goals of this project is to train undergraduate and graduate student researchers as research assistants during the academic year, both to learn relevant material to aid in research but also to develop practical skills in computational number theory and prepare for careers in academia or industry. The stipend support will be for participation in this program.

This program will provide some support to two (2) qualified undergraduates at $5000 a year each. Qualified candidates will have completed the calculus sequence at MSU or an equivalent course sequence at an accredited institution and demonstrate sufficient mathematical maturity to the satisfaction of the PI.

This program will also provide support for one (1) MSU graduate students during the academic year. The stipend support will be for work as a research assistant under the PI and also supervising undergraduate research assistants. Graduate student support at MSU is, on average, $18,000 per year.

b) Travel:

Year 1: $3,000
Year 2: $3,000
Year 3: $3,000

Annual travel funds of up to $3000 will be used to support student travel to give research or poster presentations at related conferences and to participate in workshops related to this research program. The conference locations are TBD and may or may not include the specific locations given in the PI's proposed travel plans.

G. Other Direct Costs:

a) Materials and Supplies:

Year 1: $8,300
Year 2: $3,300
Year 3: $3,300

In just the first year, $5000 will be used for the program to purchase four laptop computers for the use of the PI and each of the three student research assistants.

In each of the three years: $3,000 will be spent on general supplies (books, papers, inks, etc) to be used by the PI and the student assistants. $300 will be spent on relevant journal subscriptions.

b) Publication/Documentation: None

c) Consultant Services: None

d) Computer Services:

Year 1: $999
Year 2: $999
Year 3: $999

The PI plans on training student research assistants in SageMath to aid in the research program. For this, the PI is requesting funds for a professional subscription to CoCalc, the SageMath-powered cloud computing and project management service. This service allows for easy project sharing within the group and with remote collaborators, is based on SageMath, and affords powerful cloud computing support. The professional package costs $999 a year.

e) Sub-awards: None

f) Other: None

H. Total Direct Costs (A Through G)

Year 1: $58,890
Year 2: $55,392
Year 3: $55,195

I. Indirect Costs:

Year 1: $14,224
Year 2: $12,440
Year 3: $12,339

Using DHHS negotiated rates; the total direct cost base (A+B+C+E+G) was multiplied by the negotiated rate of 51% to obtain the indirect cost.

J. Total Direct and Indirect Costs:

Year 1: $73,114
Year 2: $67,832
Year 3: $67,534

Total Budget: $208,480

Current and Pending Support

(See PAPPG Section II.C.2.h for guidance on information to include on this form.)

The following information should be provided for each investigator and other senior personnel. Failure to provide this information may delay consideration of this proposal.

Investigator: Thomas Hulse

Other agencies (including NSF) to which this proposal has been/will be submitted.

Support: ☐ Current ☒ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title: Excellence in Research: Shifted Convolution Sums for The Congruent Number Problem

Source of Support: NSF

Total Award Amount: $ 208,480 Total Award Period Covered: 07/01/18 - 06/30/21

Location of Project: Morgan State University, Mathematics Department

Person-Months Per Year Committed to the Project. Cal:0.00 Acad: 0.00 Sumr: 2.00

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Sumr:

Support: ☐ Current ☐ Pending ☐ Submission Planned in Near Future ☐ *Transfer of Support

Project/Proposal Title:

Source of Support:

Total Award Amount: $ Total Award Period Covered:

Location of Project:

Person-Months Per Year Committed to the Project. Cal: Acad: Summ:

*If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.

Facilities, Equipment, and Other Resources

Morgan State University

The Principal Investigator, Dr. Thomas A. Hulse, is an Assistant Professor of Mathematics at Morgan State University, which is the supporting institution in this grant proposal. Morgan will mainly provide office and classroom space for work, meetings, and seminars. In addition they will provide library resources which will mean access to books, journal articles and relevant websites. Morgan also provides access to undergraduate and graduate students who may want to participate in math research projects, either within the auspices of the Math Department's Senior Seminar or outside of it, and work as research assistants with the PI. Morgan will also make all travel arrangements for Dr. Hulse and his research assistants pertaining to this proposal.

Additionally, as a new faculty member at Morgan, Dr. Hulse will also benefit from the Junior Faculty Mentoring Plan in which he will be able to draw on support from a senior departmental mentor for help supervising research and managing this grant, if necessary.

Letters of support from the dean of the School of Computer, Mathematical, & Natural Sciences, Dr. Hongtao Yu, and the chair of the mathematics department, Dr. Asamoah Nkwanta, at Morgan are included in the Program Description, as instructed in the NSF 18-522 guidelines for Excellence in Research individual investigator projects.

Unfunded Collaborators

Dr. Hulse has long-standing collaborative relationships with:

- Dr. Chan Ieong Kuan, Ph.D,
  Associate Professor,
  Sun Yat-Sen University (Zhuhai), China
- Dr. David J. Lowry-Duda, Ph.D,
  Research Associate,
  Mathematics Institute at the University of Warwick, UK
- Dr. Alexander W. Walker, Ph.D,
  Visiting Assistant Professor,
  Brown University, Providence, RI, USA

These individuals have already contributed significant thought and time to the beginnings of this project and will continue to do so as it develops, though they are not pursuing funding from this grant. Their contribution to this project will not interfere with their other professional responsibilities. Likely all papers that come out of this project will share co-authorship with them. Find letters of support from each of them in the supplementary documentation section.

Data Management Plan

As with most mathematics proposals, a significant data management plan is not required. Most data generated by programming scripts in SageMath will be temporary investigative tools for conjecture and the means to the end of proving theorems, not usually meant to be the finished product. Mainly computational work is meant to be exploratory and also serve as a tool for educational development. When this kind of computational work produces a data set that has merit for publication, it will be submitted for publication. Otherwise, all past SageMath work will be saved in the CoCalc system and can be retrieved by the PI as necessary.

All significant mathematical results will be submitted for publication to peer-reviewed journals in a timely way and all finished products will be deposited in the designated NSF repository. Results will also be presented a conferences, seminars, colloquia, and the like.

Graduate Student Mentoring Plan: This project will recruit one Morgan State University graduate student as a research assistant. The graduate student must have completed the graduate-level courses in abstract algebra and complex analysis or else be committed to completing those courses within a year. MSU currently has about eight masters students and twelve Ph.D students to recruit from. This should be the same student for the full three-year duration of the project. Ideally, this student will be the PI's thesis advisee and so portions of the project would inspire the student's thesis work. As part of training, the student will be educated in analytic number theory, the SageMath computer algebra system, and cryptography. The student will also be supervised in developing skills to aid in academic research: namely finding, reading, and communicating articles, preparing papers, giving presentations, and overseeing the undergraduate research assistants.

The primary aim of this assistantship is for the graduate student to make a meaningful contribution to the research project and, in the process, for the student to prepare for a career in academia or industry. In addition to the average graduate student stipend, funding is requested to help send the student to conferences and workshops to present their work and to make professional networking connections. The PI will guide the student through their degree program and provide career oversight by recommending events and activities for professional development as well as connecting them with resources to effectively apply for jobs. Success of this plan will be evaluated by the student's progress in their degree program, their contribution to the project, their presentation of the work at conferences or symposia or as published works in peer-reviewed journals, and significant career development.

Undergraduate Student Mentoring Plan: This project will recruit two Morgan State University undergraduate students as research assistants. These students will be chosen among those that have successfully completed the calculus sequence at MSU and have demonstrated significant mathematical maturity. This will most likely be one of the 10-15 math majors in the department each year, or alternately students may be recruited from CS, engineering or physics. The positions will be advertised in the math club, senior seminar, the weekly problem-solving sessions, in-class announcements and elsewhere. Ideally these students will also be working with Dr. Hulse as part of the MSU math department's "Senior Seminar" course, in which undergraduate math majors work on independent research projects under the guidance of faculty members. These projects culminate with a paper and a presentation at MSU's annual symposium.

As part of training, the students will be educated in number theory, the SageMath computer algebra system, and cryptography. The students will also be supervised in developing skills to aid in academic research: namely learning to read and summarize academic papers provided by the PI, learning how to use the LaTeX typesetting system, and how to give presentations. The students' primary research responsibilities will be computational work in SageMath, which will be supervised by the PI and the graduate research assistant.

The primary aim is for the students to make meaningful contributions to the research project and develop skills that will successfully prepare them for careers in academia or industry. In addition to a stipend, funding is requested to help send the students to conferences and workshops to present their work, and to make professional networking connections. These students will receive academic guidance from the PI to aid in their completion of their undergraduate program, and furthermore the PI will also provide some oversight over career planning: recommending events and activities for professional development and connecting them with resources to more effectively apply for jobs or graduate school. Success of this plan will be evaluated by the students' progress in their undergraduate program, their contribution to the project, and significant career development.

February 28, 2018

To whom it may concern,

If the proposal submitted by Dr. Thomas A. Hulse entitled "Excellence in Research: Shifted Sums for The Congruent Number Problem" is selected for funding by NSF, it is my intent to collaborate and/or commit resources as detailed in the Project Description or the Facilities, Equipment and Other Resources section of the proposal.

Sincerely,

Chan Ieong Kuan
School of Mathematics (Zhuhai)
Sun Yat-sen University
Zhuhai, Guangdong, 519082
People's Republic of China

E-mail: kuanchi3@mail.sysu.edu.cn

THE UNIVERSITY OF
WARWICK

The University of Warwick
Coventry, CV4 7AL
United Kingdom

February 28, 2018

To whom it may concern,

If the proposal submitted by Dr. Thomas A. Hulse entitled "Excellence in Research: Shifted Sums for The Congruent Number Problem" is selected for funding by NSF, it is my intent to collaborate and/or commit resources as detailed in the Project Description or the Facilities, Equipment and Other Resources section of the proposal.

With my regards,

David Lowry-Duda
Warwick Mathematics Institute
University of Warwick
Coventry, CV4 7AL
United Kingdom

Tel: +44(0) 7484 118526
Email: d.lowry@warwick.ac.uk
Web: http://davidlowryduda.com

BROWN

Department of Mathematics

February 27, 2018

To whom it may concern,

If the proposal submitted by Dr. Thomas A. Hulse entitled "Excellence in Research: Shifted Convolution Sums for The Congruent Number Problem" is selected for funding by NSF, it is my intent to collaborate and/or commit resources as detailed in the Project Description or the Facilities, Equipment and Other Resources section of the proposal.

Sincerely,

Alexander Walker
Department of Mathematics
Brown University

Email: alexander_walker@brown.edu
Web: awwalker.com