Final Report

Improving Public School Bus Operations Boston and Baltimore County Public Schools

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16. Abstract Studies show that congestion in big cities has a tremendous impact on the time travelers spend on the road. This is translated into a loss of productivity and also impacts students relying on school buses to commute to their schools. In fact, a common problem facing schools is students arriving late for breakfast and/or classes. The objective of this research is to develop a system that allows the Boston Public Schools (BPS) and Baltimore County Public Schools (BCPS) to transport students to and from schools in a safe, reliable, and optimum manner. Due to BPS and BCPS's system of school choice and geography, some students need to travel long distances to attend school. This problem is complex and has many dimensions, and we built a system that uses historical and real-time traffic data to predict the traffic state evolution over a short time horizon. This is then coupled to an advanced routing algorithm to route buses in an optimal fashion to improve the quality of service.		
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Abstract

Studies show that congestion in big cities has a tremendous impact on the time travelers spend on the road. This is translated into a loss of productivity and also impacts students relying on school buses to commute to their schools. In fact, a common problem facing schools is students arriving late for breakfast and/or classes. The objective of this research is to develop a system that allows the Boston Public Schools (BPS) and Baltimore County Public Schools (BCPS) to transport students to and from schools in a safe, reliable, and optimum manner. Due to BPS and BCPS's system of school choice and geography, some students need to travel long distances to attend school. This problem is complex and has many dimensions, and we built a system that uses historical and real-time traffic data to predict the traffic state evolution over a short time horizon. This is then coupled to an advanced routing algorithm to route buses in an optimal fashion to improve the quality of service.

1 Introduction

Traffic congestion has become an everyday problem in many urban areas, bringing with it negative environmental impacts. During periods of congestion, cars and public transportation cannot run efficiently, resulting in air pollution, carbon dioxide (CO_2) emissions, and increased fuel use. In 2007, wasted fuel and lost productivity cost Americans $87.2 billion. This number reached $115 billion in 2009 [1]. Congestion also increases travel time. For delivery companies and local bus transits systems, this is a source of delay, increased costs, and customer dissatisfaction.

Boston Public Schools (BPS) owns and operates a fleet of approximately 700 school buses that transport students throughout the Greater Boston area to their respective schools. On a typical day, an estimated 27,000 students are driven across the city to approximately 230 school locations [2]. For this logistical challenge to succeed, approximately 3,000 individual bus trips are needed. Since the students live "scattered" across the city, the buses, during rush hour, cover nearly 45,000 miles of almost all road types and, naturally, congestion levels. Equipped with a Global Positioning System (GPS) tracker, these buses are monitored in real time (i.e., position and velocity). BPS therefore has rich data that can be exploited to provide a live and accurate picture of the traffic through the city of Boston.

The logistical problem of delivering goods and/or people from starting locations to destinations and/or multiple destinations is not new. In fact, it is very similar to the bike share-rebalancing problem (BSRP) [3-5] and the problem of vehicle scheduling [6]. Given a number of bike stations scattered around a city, a truck or a group of trucks loops through to pick up excess bikes or drop off a number of needed bikes at each station. This operation is usually performed during the night, and thus does not face many constraints and is not time sensitive. However, in the bus routing problem, the operations need to be performed within a specific time frame since students need to arrive on time at schools to attend their classes. Given the ever-changing traffic patterns in big cities, the latter objective is challenging. The first solution schools adopt to improve on-time performance is to increase the number of buses, split the routes, and reassociate bus stops to the new routes. This is effective at reducing delay, but at the same time raises costs, increases the environmental impact, and adds slightly to the existing congestion.

Previous attempts in Boston to optimize the system resulted in an improved on-time performance while at the same time reducing the number of buses and operating costs. Using a smaller number of buses, namely 600, approximately 71% arrived at their schools before the opening bell, which resulted in savings of nearly 5 million dollars [2]. This attempt had a welcomed side effect: $CO2$ emissions were reduced by 40,000 pounds.

Achieving higher on-time performance is expected to be challenging. This is partly due to different factors. These factors include the following:

1. Maximum number of students a bus can pick up;
2. The length of the route;
3. The scheduled number of stations for pickup and their locations;
4. The time of day;
5. The desired time spent on the bus for the different students.

The School Bus Routing Problem (SBRP) discussed this report is essentially a complicated optimization task. In this problem, a bus is expected to take students from different locations (i.e., bus stops) and drop them at different destinations (i.e., different high, middle, and elementary schools) in one single trip. Improving this process is expected to reduce delay times (i.e., increase the on-time performance) and ultimately improve the level of service.

2 Literature Review

The SBRP has been considered since the 1970s. The main drive for this effort is to improve the efficiency, economic performance, and quality of service for students. SBRP is very similar to the Vehicle Routing Problem (VRP) [7-11]. Given a fleet of vehicles of unknown size (i.e., number), different stations for the pickup and drop-off of goods or passengers, and a network of roads, the objective is to determine the optimum number of vehicles traveling throughout the network in optimal routes to serve the stations (i.e., alighting/boarding or delivering/picking up at the same time). For SBRPs, there are additional constraints. For instance, buses need to pick up the students within a specific time frame. The total vehicle traveling time is also constrained.

The first attempts to develop solution algorithms to the SBRP were conducted between the early 1970s and the 1990s. During that period, most studies focused essentially on serving a single or multiple schools and a limited number of constraints. Bennett and Gazis [12] introduced the SBRP with a single school. Their proposed model essentially aimed to minimize the total student traveling time, and bus capacity was the main constraint of the model. At the same time, Angel, et al. [13] proposed a school bus scheduling algorithm minimizing the number of routes, vehicle travel distance, bus loads, and traveling time for each route. Bus capacity and traveling time limitation for some routes were the main constraints of the algorithm. Later, Bodin and Berman [14] extended this algorithm by adding a routing capability, considering minimization of total bus travel as the goal of algorithm with a travel time window for the students. Newton and Thomas [15] solved an SBRP by considering multiple schools and minimizing the total bus travel time as well as the number of routes, with student travel time and bus capacity the main constraints of their model.

Dulac, et al. [16] developed a single-school SBRP model aimed at minimizing the total number of buses, routes, and total travel distance by considering the number of stops, length of routes, and fleet capacity as constraints. Bowerman, et al. [17] developed a more advanced version for the single-school multi-objective SBRP. The aim was to minimize the number of routes, trip lengths, bus loads, and walking distance of students while considering bus capacity and the total travel time. Finally, in order to determine the optimum fleet size, Braca, et al. [18] solved a multi-school, multi-objective SBRP for New York having constraints on bus capacity, student travel distance, and school time window.

In the 2000s, more advanced models with sophisticated constraints were elaborated for the SBRP. Li et al. [19], provided an algorithm that minimizes the total number of buses and travel time for students. They added a new constraint: the earliest pick-up time of students.

They used a heuristic method to transform the problem into a Traveling Salesman Problem (TSP). Schittekat, et al. [20] in their work tried to minimize the students' walking distance and the school bus travel time by finding the optimum spots where the students can walk to be picked up. More comprehensive developed algorithms can be found in [21, 22].

3 Purpose of the Study

The objective of this research effort is to improve our understanding of traffic flow throughout the City of Boston. Using historical data gathered by BPS, we will attempt to develop traffic prediction and bus routing tools.

These tools will help BPS estimate the short-term evolution of the traffic state and can ultimately be combined with an optimization tool (i.e., multi-objective advanced routing algorithm) that gives recommendations and detailed bus routes. This information would be updated in real-time to adapt the routes and increase the on-time performance of the fleet while at the same time reducing the operational costs.

Even though SBRP algorithms have been in development since the 1970s, there are a few aspects that have not been treated previously. One of these aspects is the degree of circuity (DOC). DOC is defined as the ratio of the student in-vehicle travel time to the direct distance between the student's house and the school. A major complaint of parents is that their children experience long travel times to the schools when they live very close. To address this issue, we add the DOC ratio as an upper bound constraint on the traveling time for all students. This mandates that students must be served within an allowed maximum time duration.

Another aspect of interest is the interconnectivity of the time windows to serve the various schools and the potential benefit sought when serving these institutions concurrently. Specifically, school buses leave the depot in the early morning to pick up high school students and deliver them to their designated high schools. The buses then leave these high schools to pick up middle school students and deliver them to the middle schools. After that, the buses leave to pick up elementary school students to drop them off at their

respective elementary schools. Clearly, this procedure is not making efficient use of the available resources. One natural solution is to pick up the different students concurrently (i.e., within the same trip) and transport them to their respective drop-off locations.
This research develops two innovative contributions to the optimal algorithm. First, and with consideration of the nature of the school bus operation, it contains multiple levels of schools in a single trip. It also features multilevel optimization in one single framework. Second, maximum travel circuity for each student is applied to improve the level of service.

4 Accurate Estimation of the Travel Time on the Greater Boston Area Road Network

In the Boston area, it is estimated that on a typical day 27,000 students are driven across the city to approximately 230 school locations [23]. To achieve this goal, 3,000 individual trips are needed. Since the students live "scattered" across the city, the buses cover nearly 45,000 miles of almost all road types and congestion levels. With this large scale, congestion and inefficiencies in the bus transportation system have an important impact on the on-time performance.

Aware of this challenge, BPS is working on re-planning the school bus routes to minimize the congestion buses experience in the network and ensure the safe and optimum transportation of schoolchildren.

BPS uses Versatrans, a planning software that helps public schools design school bus routes that minimize congestion and travel times. One important piece of input needed by Versatrans is the travel time/speed on the various roads of the network. Therefore, an accurate estimation of the link travel time/speed is a prerequisite.

Our objective in this section is to improve our understanding of traffic flow throughout the City of Boston. Using historical data gathered by BPS, we will estimate the link travel times and adapt/enhance the routes to increase the on-time performance of the fleet while at the same time reducing the operational costs.

4.1 Data Cleaning and Preparation

The data sets used in this study include two sources: the bus route planning data provided by Versatrans and the GPS data collected by buses operating in the area.

The planning data are in the numeric format of the Boston road network system. The road links are recorded as multiple segments. In total, there are 688,190 road segments. Each segment has the following major attributes:

- Start Point Latitude
- Start Point Longitude
- End Point Latitude
- End Point Longitude
- Speed Class
- One Way

The GPS data collected by the buses covers almost all the road types in Boston and all of its neighborhoods, providing a realistic representation of the traffic. The data were collected over the course of several months. The school bus dataset has the following major attributes:

- Log time
- Longitude
- Latitude
- Heading
- Speed
- Bus ID

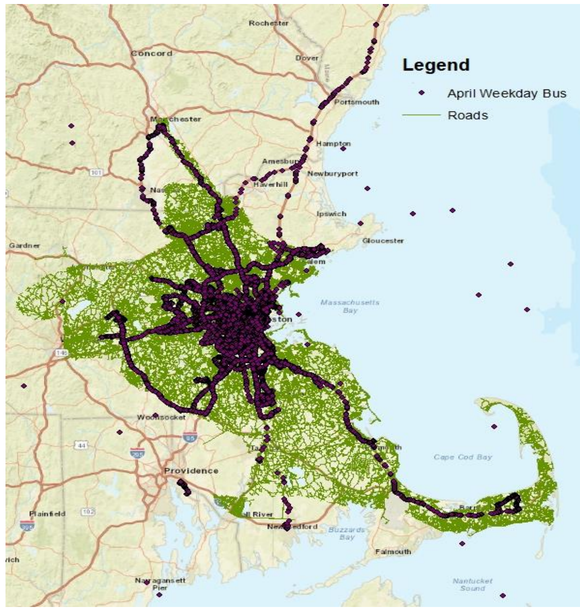

Figure 1: Bus data points and Boston area

To prepare the data for our analysis, the road network in the Boston area was geocoded in ArcGIS from the Versatrans planning data. Meanwhile, the school bus data were geocoded in ArcGIS based on the longitudes and latitudes. The two datasets are illustrated in

. As can be seen, the bus routes cover a large area of Boston and some even extend outside the state.

4.2 Modeling Algorithm

The goal of our model is to estimate travel speeds for road links for better planning of the bus routes with minimum delay. A preliminary examination of the bus GPS data reveals that the errors in the GPS speeds are dramatic. The data delivers the instantaneous speed at a specific time, but the acceleration and deceleration skew the traveling speed significantly, especially when the density of traffic signals or other traffic control signs is high. Another challenge is that the device used onboard the buses to collect location and speed data had a low accuracy level. Thus, a significant amount of the collected data has recorded speeds of zero.

To address the issue of zero speed, we propose a model to estimate the spatial mean traveling speeds for road links. Instead of the speed data recorded by the buses, the timestamps and location information of the buses were used in the speed estimation. To avoid bias and to ensure that the estimation was as accurate as possible, we only included weekday data in April. The speeds for this data are typical of what school buses would experience on weekdays. Once the target data were extracted from the raw data, they were spatially related to the road network database. This was performed by linking each recorded point (via the bus GPS device) to the closest road segment. A group of data points was associated with each road segment/link and examined for the following criteria: (1) Is the block of points associated with more than one bus ID; (2) Is there a time gap that is too large between consecutive points; and (3) Did the traveling direction change. If the answer to any of the above questions is positive, it may indicate that a trip ended in the middle of the link and therefore the block of points was split into two blocks. The flowchart of the data processing steps is illustrated in Figure 2.

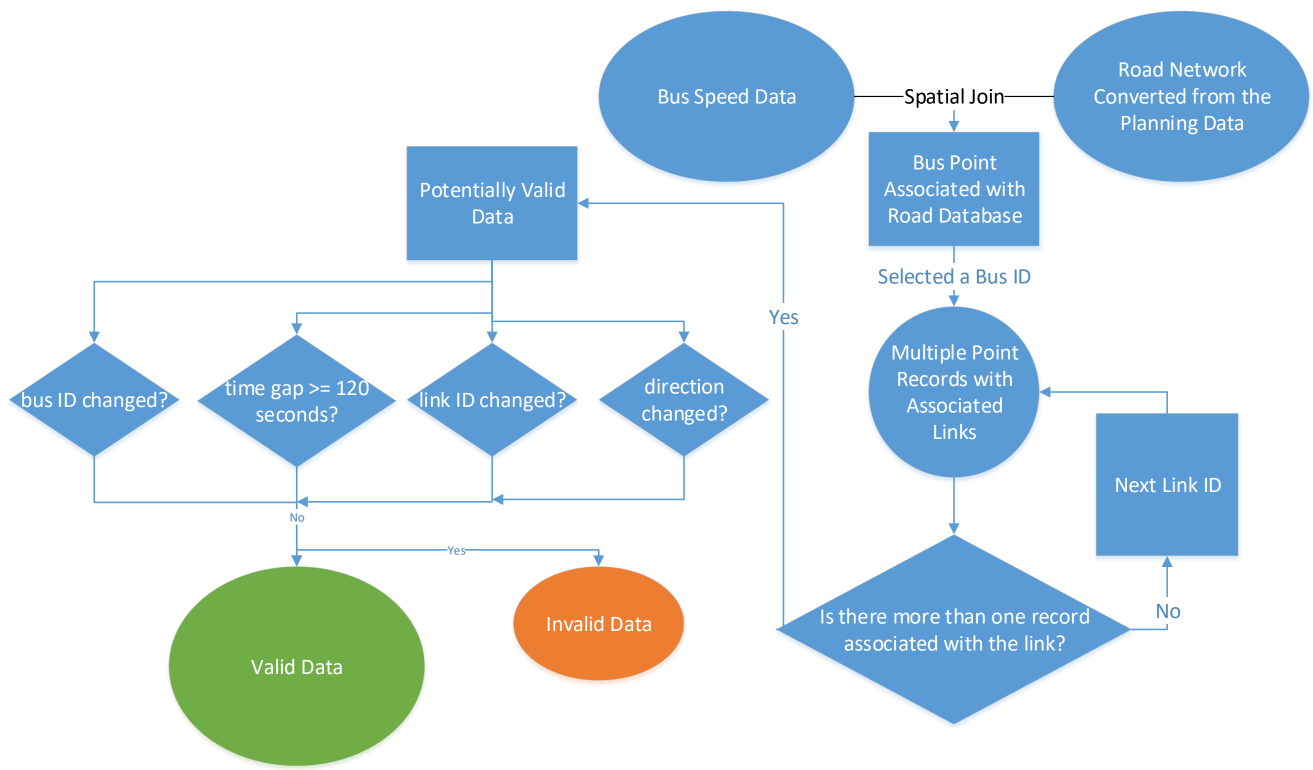

Figure 2: Data processing flowchart

The time difference between the very first point traveled on a link and the last point during the same trip was calculated. Simultaneously, the latitudes and longitudes of these two points were used to calculate the distance traveled using Equation (1). A space mean traveling speed was then determined for the link.

$$D = 3961 * Arccos(\cos(90 - lat1) * \cos(90 - lat2) + \sin(90 - lat2) * \sin(90 - lat1) * \cos(long2 - long1)) \quad (1)$$

Due to the directional and temporal variation of travel speeds in the Boston area, we also categorized the data by the time of day to help facilitate the bus route planning process during different times of day when the traffic congestion varies. The categories for the time cut lines are listed in Table 1. Meanwhile, we set the minimum observations on one link to be three to avoid outlier bias (i.e., a link needs to have a minimum of three records to be valid). The median of all the calculated space mean speeds was used to represent the link travel speed. After the median link travel speed was generated, the speeds were color-coded and visualized in ArcGIS. The morning and afternoon peak link travel speeds are illustrated in Figure 3.

Table 1: Times for Different Planning Periods

Morning Peak	7 a.m. – 10 a.m.
Midday	2 p.m. – 4 p.m.
Afternoon Peak	4 p.m. – 7 p.m.
Night	7 p.m. – 7 a.m.

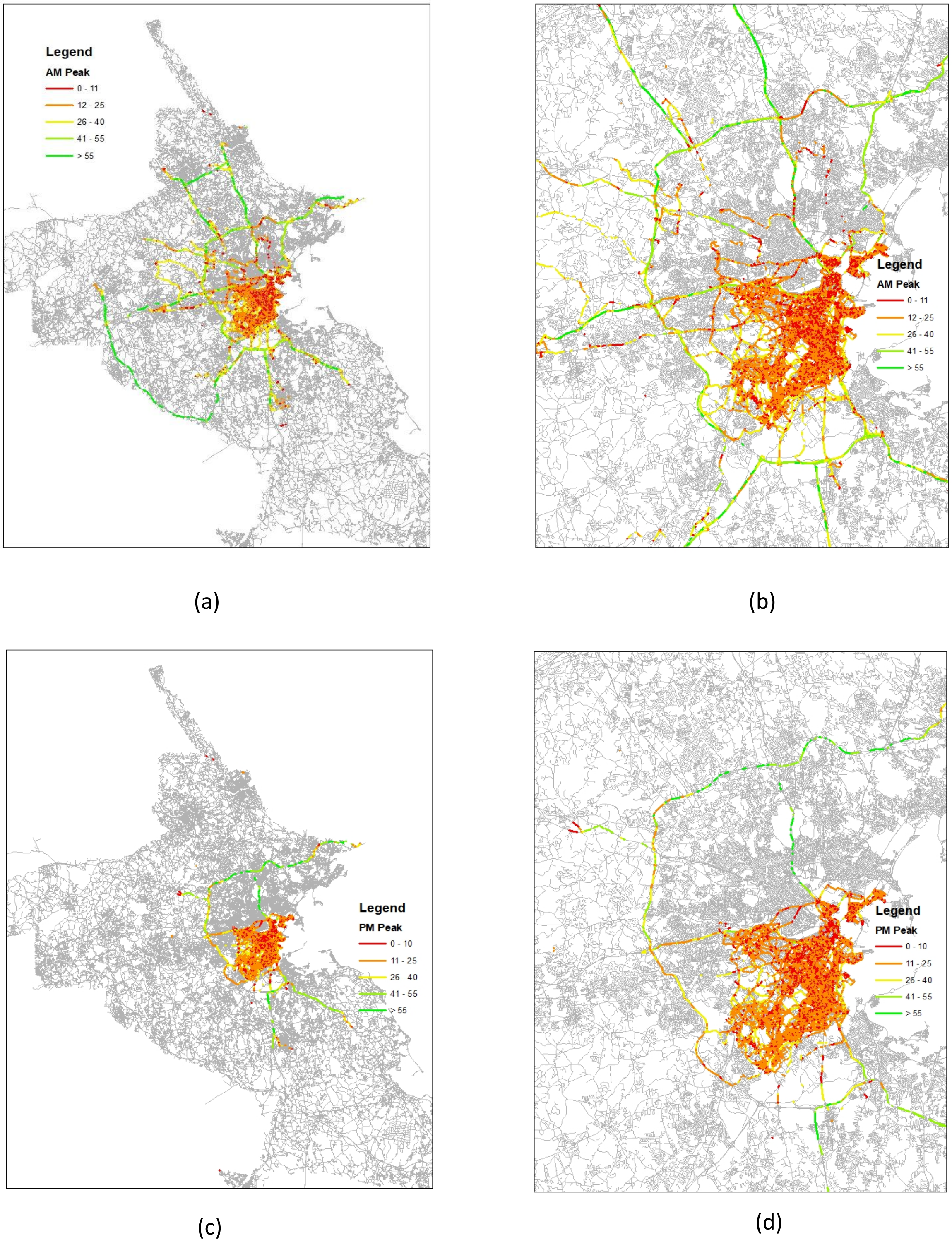

Figure 3: Travel speed: (a) a.m. peak; (b) zoomed in a.m. peak; (c) p.m. peak; (d) zoomed in p.m. peak

4.3 Application of the Model

BPS is responsible for the design of bus routes and stops for school buses. They use Versatrans to plan the routes. One of the prerequisites for this software to deliver accurate and relevant results is the estimated travel time for each road link. The existing travel times are supposedly provided over three time periods: morning peak, afternoon peak, and the rest of the day. BPS uses the travel time data to generate routes for school buses for different time frames.

However, a close examination of the collected data shows various anomalies:

1. The majority of the data is missing. Of the 688,000 road links, 460,000 have "null" values and 220,000 have zero velocity.
2. For the road links that do have travel time data, the values for all three time periods are mostly the same, meaning that no dynamics or evolution in travel speed is considered in the database.

Our intuition suggests that instead of taking variation into account in the database, the travel times are estimated by simply computing distance over the speed limit. This crucial and missing variation in travel speed is the source of inaccuracies, and therefore the routes generated based will result in transportation inefficiency. In other words, the software is fed the wrong data and generates non-relevant routes to the problem. This poor-quality dataset will not only result in delays in traffic but possibly cause more congestion and impact other road users.

To create more-efficient bus routes, a more-accurate travel time database for the whole network is needed. Specifically, the time needed to traverse each road link at different times of the day should be known a priori.

The model developed in this study uses the real-time travel trajectories of the school buses to make spatial inferences for the link travel times. Since the input data recorded the timestamps of the trajectories of the buses, the estimated travel time can reflect the temporal and spatial variation of the travel speeds by time of day. Therefore, they are more accurate in describing the traffic condition in the network. Using the outputs of our model, BPS can create a more reasonable school bus route system to bypass congested areas and minimize the time that school children need to spend on buses.

5 Development of a Routing Algorithm for Buses

In this section, we propose an algorithm that determines the best route for a series of buses that pick up students and drop them off at their respective schools (i.e., elementary, middle, and high schools).

5.1 Proposed Algorithm

In contrast with VRPs, the time value of students is not the most important object in most school bus problems; however, the school bus problem must serve all the students in the defined time windows. Therefore, the main objective of the problem is minimizing the total costs of the school bus transit system, including the operating cost of the school bus service operator and traveling time of the students.

The proposed problem is multiple school bus routings consisting of a single depot and multiple schools at the high school, middle school, and elementary school levels. The problem consists of defining less costly routes to visit exactly once the boarding locations in the morning time windows and the alighting locations in the afternoon time windows were determined. In the morning time window, each route must start at the depot and connect—in order—to the high school, middle school, and elementary school and finish at the depot. The school buses serve all students and never violate the vehicle capacity and time window constraints. The mathematical formulation is as follows:

Parameters:

$\boldsymbol{I_s^t}$**:** Number of students of school **s** of (t = 1: Elementary school, t = 2: Middle school, t = 3: High school)

$\boldsymbol{o_k}$: origin of bus **k** in morning

$\boldsymbol{K}$: total number of buses

$\boldsymbol{n^t}$: number of schools (t = 1: Elementary school, t = 2: Middle school, t = 3: High school)

$\mathbf{d_{ij}^s}$ = direct distance between the home of student i and the home of student **j** of school **s**

$\mathbf{d_{io_k}^s}$ = direct distance between the home of student **i** and school **s** and the start location of bus **k**

$\mathbf{dis_{i0}^s}$ = direct distance between student **i** of school **s** and location of school **s**

Speed: bus speed

Timeratio: maximum allowed ratio for students (in bus trip time/direct time to school)

cycle_time = 60 minutes; allowed time for transferring each of middle school, high school, and elementary students

C = capacity of buses

M: a big enough number

$\mathbf{C_T}$: time value of each student per hour

$\mathbf{C_O}$: unit operating cost of school bus per kilometer

Variables

$$v_k^s = \begin{cases} 1 & bus\ k\ is\ used\ to\ transfer\ students\ of\ school\ number\ s \\ 0 & otherwise \end{cases}$$

$$y_{ik}^s = \begin{cases} 1 & bus\ k\ is\ used\ to\ transfer\ student\ i\ of\ school\ number\ s \\ 0 & otherwise \end{cases}$$

$$\alpha_{ijk}^s = \begin{cases} 1 & school\ s\ student\ j\ is\ picked\ up\ (dropped\ off)\ after\ student\ i\ in\ bus\ k \\ 0 & otherwise \end{cases}$$

$$\alpha_{i0k}^s = \begin{cases} 1 & the\ bus\ k\ goes\ to\ destination\ after\ school\ s\ student\ i\ is\ picked\ up\ (dropped\ off) \\ 0 & otherwise \end{cases}$$

$$\alpha_{i0k}^{s} = \begin{cases} 1 & bus\ k\ starts\ from\ origin\ before\ school\ s\ student\ i\ is\ picked\ up\ (dropped\ off) \\ 0 & otherwise \end{cases}$$

$D_{ik}^{s} = distance\ travelled\ up\ to\ student\ i\ of\ school\ s\ by\ bus\ k$

$TotalD_k = total\ distance\ travelled\ by\ bus\ k$

$AT_i^s = Vehicle\ arrival\ time\ to\ the\ home\ of\ student\ i\ of\ school\ \ s$

$WT_i^s = In\text{-}vehicle\ travel\ time\ of\ student\ i\ of\ school\ s$

$UC_k\ = used\ capacity\ of\ bus\ k$

Objective function

$$z = \min \sum_{s=1}^{n^t} \sum_{i=1}^{I_s^t} C_T\ * WT_i^s + \sum_{k=1}^{K} C_O *\ TotalD_k \tag{1}$$

Constraints

$$\sum_{k=1}^{K} y_{ik}^{s} = 1 \qquad i = 1, 2, \dots, I_s^t\ \ s = 1, \dots, n^t \tag{2}$$

$$\sum_{i=1}^{I_s^t} y_{ik}^{s} \leq Mv_k^s \qquad k = 1, 2, \dots . K, s = 1, \dots, n^t \tag{3}$$

$$\sum_{s=1}^{n^t} v_k^s \leq 1 \qquad k = 1, 2. \dots, K \qquad (4)$$

$$2 * (\alpha_{ijk}^s + \alpha_{jik}^s) \leq (y_{ik}^s + y_{jk}^s) i, j = 1, 2, \dots . I_s^t; i \neq j;\ k = 1, 2 \dots, K, s = 1, \dots, n_t \qquad (5)$$

$$\sum_{k=1}^{K} \sum_{j=1}^{I_s^t} \alpha_{ijk}^s + \sum_{k=1}^{K} \alpha_{i0k}^s \geq 1 \qquad i = 1, , \dots, I_s^t \qquad s = 1, 2, \dots, n^t \qquad (6)$$

$$\sum_{\mathrm{k}=1}^{K} \sum_{i=1}^{I_s^t} \alpha_{ijk}^s + \sum_{k=1}^{K} \alpha_{0jk}^s \geq 1 \qquad s = 1, 2, \dots, \mathrm{n}^{\mathrm{t}}, \qquad j = 1, 2, \dots, I_s^t \qquad (7)$$

$$\sum_{j=1}^{I_s^t} \alpha_{0jk}^s = v_k^s \qquad s = 1, \dots, n_t \qquad k = 1, 2 \dots, K \qquad (8)$$

$$\sum_{i=1}^{I_s^t} \alpha_{i0k}^s = v_k^s \qquad s = 1, \dots, n_t \qquad k = 1, 2 \dots, K \qquad (9)$$

$$D_{jk}^s \geq D_{ik}^s - M\left(1 - \alpha_{ijk}^s\right) + d_{ij}^s \qquad s = 1, \dots, n_t \quad i \& j = 1, 2, \dots, I_s^t; i \neq j \quad k = 1, 2 \dots, K \ (10)$$

$$D_{ik}^s \geq d_{0_k i}^s - M\left(1 - y_{ik}^s\right) \qquad s = 1, \dots, n_t \quad i = 1, 2. \dots . I_s^t; i \quad k = 1, 2 \dots, K \qquad (11)$$

$$AT_i^s \geq \sum_{k=1}^{K} \frac{D_{ik}^s}{speed} \qquad s = 1, \dots, n_t \quad i = 1, 2, \dots, I_s^t \qquad (12)$$

$$WT_i^s = \frac{\mathrm{TotalD_k}}{speed} - AT_i^s \quad i = 1, 2. \dots . I_s^t \quad s = 1, \dots, n_t \tag{13}$$

$$WT_i^s \leq Timeratio * \frac{dis_{i0}^s}{speed} \qquad s = 1, \dots, n_t \quad i = 1, 2, \dots, I_s^t \tag{14}$$

$$AT_i \leq DOC * \frac{d_{i0}}{speed} \qquad i = 1,2. \dots . I \tag{15}$$

$$AT_i^s + \frac{dis_{i0}^s}{speed} \leq \text{cycle time} \qquad s = 1, \dots, n_t , \quad i = 1, 2, \dots, I_s^t \tag{16}$$

$$UC_k = \sum_{s=1}^{n^t} \sum_{i=1}^{I_s^t} y_{ik}^s \leq \mathrm{C} \qquad k = 1, 2, \dots, K \tag{17}$$

$$\mathrm{TotalD_k} \geq D_{ik}^S + dis_{i0}^s - M\left(1 - y_{ik}^s\right) \qquad s = 1, \dots, n_t , \quad i = 1, 2, \dots, I_s^t \tag{18}$$

$$v_k^s = (0.1), \ \alpha_{ijk}^s = (0.1), \ y_{ik}^s = (0.1)$$

$$D_{ik}^s \geq 0, \ TotalD_k \geq 0, \ AT_i^s \geq 0, \ WT_i^s \geq 0, \ UC_k \geq 0$$

Formula (1) is the objective function of the problem. It tries to minimize the waiting time of all students and total driven distance. Constraint (2) specifies that each student of each school is served by exactly one bus. Constraint (3) ensures that if a student is assigned to a bus, the related bus is considered as a used bus for that school. Equation (4) makes sure that each bus is only assigned to one school. Equations (5), (6), and (7) define the path of each bus. Equations (6) and (7) ensure that each passenger is assigned to a path. Zero is the index used for the origin/destination. Each path starts from an origin and ends at the destination (for morning, the destination of the bus is the related school). Equations (8) and (9) ensure that every route starts from the origin and ends at the school (for morning), and multiple trips of the bus are not allowed. Equations (10) and (11) calculate total travelled distance up to student **i** of school **s**. For the first picked-up student, the total travelled distance of the bus is the direct distance of the bus's origin to the student's home.

Equation (12) defines the arrival time of the bus to students. Equation (13) calculates in-bus waiting time for students, and Equation (14) is a time ratio constraint. It ensures that the total waiting time of each student is less than the direct travel time of the student to the school multiplied by the direct index. Constraint (14) is an additional time ratio constraint, and Constraint (16) is the cycle time constraint that ensures all students will be at school at the desired time. Equation (17) is a capacity constraint. Total travelled distance is defined by Equation (18).

In order to solve the problem in the proposed mode, a Simulated Annealing (SA) algorithm, one of the most efficient metaheuristic approaches for solving VRP and TSP problems, was applied [24-26]. The local search strategy of the SA method allows the algorithm to escape from local minima and jumps in the solution space to find a global optimum solution. Although the computation times of SA may be longer than other efficient and successful metaheuristics for VRPs, SA considers all possible answers, even those that may provide non-optimal solutions. Therefore, this algorithm searches for the optimal solution in a wider space that is suitable for solving permutation problems like VRPs and TSPs [27]. The SA metaheuristic approach has been widely used in similar Vehicle Routing Problem with Time Windows (VRPTW)-based algorithms [28-31].

The developed algorithm has two sub-algorithms. The first sub-algorithm defines the origins of travel for the buses, which defines how many school buses are available at the schools or at the depot. Since the travel origin of the buses in each time window is different and all-time windows must be considered in a unit framework, this sub-algorithm defines available buses at each school for each time window. For example, the travel origin of buses serving high school students in the morning time window is only a single depot; however, the travel origins of buses serving middle school students are two high schools. Therefore, this algorithm in every single stage of the framework determines the origins and destinations of buses. In other words, the algorithm dictates that a bus, after leaving an origin (for example, High School 1), should be assigned to a specific middle school (for example, Middle School 3). This assignment is based on the location of students who are assigned to the same school. In this sub-algorithm, for example, if students **i** and **j** are living near each other and they both want to go to the same school, then the bus that serves student **i** most likely serves student **j** as well. This algorithm is based on the Backward Reduction Algorithm (BRA) that calculates the distance between the original probability distribution of all possible alternatives and the probability distribution of the reduced alternatives; it then can find a near-optimal subset of alternatives given certain cardinality [32, 33]. Therefore, this sub-algorithm first creates all possible alternatives then deletes scenarios in an iterative process until it reaches the condition of a near-optimal subset. Figure 1 shows the implemented BRA to optimal assignment of origins.

Step 0: *Initialization*:

Set *s* = 1 (school index), desired_agents = J**,** I = 1 (index of student), selected_students = None,
remained_ students = All students

Step 1: *Calculate sum of distance of each school s student's home from the other students' homes*

Distance (i) = sum of the distance of student i's home from other students' homes

Step 2: IF i < I, **THEN**

I = i + 1

go to **step 1**, otherwise go to **step 3**

END IF

Step 3*: Find agents* for school s:

Step 3.1: **IF** number of students in remained_ students set < desired_agents, **THEN**

go to **step 3.2**, otherwise go to **step 4**

END IF

Step 3.2: **Find** student i from remained_students set whose home is closest to others (find the minimum of distance)

Step 3.3: **Add** i to selected_students set

Step 3.4: **Remove** i from remained_students set and go **step 3**

Step 4: **IF** s < S, **THEN**

set s = s + 1 and go to **step 1**, otherwise go to **step 5**

END

Step 5: Set s = 1, a = 1 and go to **step 6**

Step 6*: Find origin for school s:*

Step 6.0: ***Initialization***:

remained_origins = *set of available origins,* selected_origins set for school s = *Null*

Step 6.1: **IF** s <= S, **THEN**

go to **step 6.2**, otherwise go to **step 7**

END IF

Step 6.2: **IF** a <= desired_agents, **THEN**

go to **step 6.3**, otherwise go to **step 6.1**

END IF

Step 6.3: **Find** origin o from remained_origins set which is closest to the agent a's (student's) home.

Step 6.4: **Remove** o from remained_origins set and go **step 6.5**

Step 6.5: **Add** o to selected_origins set for school s

Step 6.6: Set a = a + 1 and go to **step 6.2**

Step 6.7: Set s = s + 1 and go to **step 6.1**

Step 7: End

Figure 4: The developed BRA for origin assignment

The second sub-algorithm defines the optimal routing of school buses. At first, this sub-algorithm creates a random series of integers for creating the initial solution. The algorithm allocates school buses to students depending on the location of greater integers in the generated permutation, and then, based on the order of integers in the generated permutation, the routes of vehicles are determined. For instance, in the presence of two vehicles and 10 students, a permutation of integers from 1 to 11 is produced. Suppose that the generated permutation is as follows: Path= [10, 1, 3, 4, 8, 11, 2, 9, 5, 7, 6], then the route of the first bus would be generated by serving passengers 10, 1, 3, 4, and 8, respectively, and the second vehicle route is made by serving passengers 2, 9, 5, 7, and 6, respectively. After generating an initial feasible solution ($x^{initial}$), the SA algorithm tries to create a neighboring solution by perturbing the initial solution according to the cost of the neighboring solution (the value of a hypothesized objective function which includes penalties for modeling constraints). If this cost (x^{new}) is less than that of the initial solution, then this new cost will be accepted; otherwise, it will be accepted with probability P = $e^{-\frac{\Delta f}{T}}$, where Δf is the change in cost between the new solution and previous solution and T is the initial temperature. The SA algorithm decreases the temperature during the iteration process with a controlled ratio of ($\alpha = 0.98$). In each iteration, the algorithm tries to improve the solution by searching its neighborhoods. For this purpose, the SA algorithm uses common swap, insertion, and reversion methods randomly. The algorithm attempts to reduce the value of the hypothesized objective function. The original and hypothetical objective functions are calculated as follows:

$$Z' = C_O \times Total\ vehicles\ travelled\ distance + \ C_T \times Total\ students\ in\ vehicle\ travel\ time$$

where C_O is the unit operating cost of each vehicle kilometer and C_T is the time value of each passenger per hour. Basically, the SA algorithm improves the solution by using two transformation methods, move and replace highest average, to explore the solution space. The move transformation explores groups of passengers that have the closest distances to each other including their destinations. Then it selects a random route and allocates the random passengers in the routes according to the constraints. The replace highest average transformation is based on the average distance of every group of passengers; therefore, the algorithm selects random routes and allocates selected passengers in the route in order to minimize the cost. This permutable process (cooling schedule) will end when the temperature reaches below 0.001 and the final solution does not change in iterations. Finally, the best feasible solution found during the total iterations is presented as the final solution proposed by the algorithm.

In order to consider individual students' acceptable travel times and acceptable circuity of the routing, DOC and maximum DOC (Max DOC) as shown in Equations 19 and 20 are introduced [34, 35]. The given Max DOC and computed shortest travel times are used to define the maximum acceptable travel time for each student. Using those maximum acceptable travel times for students as constraints, optimal routings are developed for each station using the SA algorithm.

$$\text{Degree of Circuity } (DOC)_i = \frac{(Actual\ travel\ time)i}{(Shortest\ travel\ time)\ i} \tag{19}$$

$$\text{Maximum Degree of Circuity (Max } DOC) \geq \max\ [DOC_i] \tag{20}$$

i = individual student

The developed model was coded in MATLAB and run on a computer equipped with an Intel Core i7 4.0 GHz (16 GB RAM) processor. Due to containing several binary variables, the model turns to a Non-Polynomial (NP)-hard problem. The pseudo code shown in Figure 2 describes the steps in the SA algorithm as applied to solve the proposed SBRP.

Step 0: ***Initialization***:

Set *s* = 1 (school index), Best Cost = *positive infinite*, T = T_0, alpha = 0.99, J = number of buses assigned to school s

Step 1: ***Create random solution***

Considering the length of trip (number of students of school s + buses of school s (*J*)-1)

set x as a random solution

Step 2: ***Find optimal solution***:

IF It1 < It1max, **THEN**

go to **step 3**, otherwise go to **step 5**

END IF

Step 3: **IF** It2 < It2max, **THEN**

go to ***step 3.1***, otherwise go to **step 4**

END IF

Step 3.1: ***Creating neighborhood***:

set x_{new} = a neighborhood of x

Step 3.2: **IF** best cost for x < best cost for x_{new}, **THEN**

set x = x_{new} and go to *step 4.5*, otherwise go to *step 4.3*

END IF

Step 3.3: p = exp-(cost x_{new} – cost x)/T*Cost x

Step 3.4: Accept x = x_{new} by p -probability and reject- and x = x_{new} by (1 - p) and go to *step 4.5*

Step 3.5: ***Cost calculation*** for x_{new}

Step 3.6: **IF** best cost for x_{new} > best cost, **THEN**

set bestsol = x_{new}

END IF

Step 3.7: ***Reducing the temperature***:

set T = alpha*T0 (0 < alpha < 1)

Step 3.8: Set It2 = It2 + 1 and go to **step 3**

Step 4: Set It1 = It1 + 1 and go to **step 2**

Step 5: **IF** bestsol is feasible, **THEN**

go to **step 7**, otherwise go to **step 6**

END IF

Step 6: Show “The problem is not feasible; more vehicles are needed.”

Step 7: **IF** s < S, **THEN**

set s = s + 1 and go to **step 1**, otherwise go to **step 8**

END IF

Step 8: ***Show results***

Step 9: **END**

Figure 5: The developed SA algorithm to solve the proposed SBRP

5.2 Examples

In order to test the algorithm, a hypothetical school bus network consisting of one bus depot, two high schools, three middle schools, and four elementary schools was developed based on the simplified school bus operation for Baltimore County Public Schools in the State of Maryland.

It is assumed that there are 260 students for each level of school, which means there are 260 students in the two high schools (H1, H2) with 130 students for each high school, three middle schools (M1, M2, M3) with about 86 students for each middle school, and four elementary schools (E1, E2, E3, E4) with 65 students for each elementary school; in total there are 780 students in the network. It is assumed that a total of 12 buses are available and that all 12 buses can run every morning and afternoon. Also, it is assumed that each bus has a capacity of 30 seats and the average speed of each bus is 30 kilometers per hour. The bus operating cost per kilometer is assumed to be 3 dollars, and the time value of each student is considered to be 10 dollars per hour.

The problem has two separate time periods, morning and afternoon. In the morning, there are three time windows for each time period, and in the first time window (6 a.m. to 7 a.m.), the school buses only serve high school students by picking them up at their location and dropping them at their designated high schools. In the second time window (7 a.m. to 8 a.m.), buses start their trips from high schools and go to middle schools, picking up only middle school students. In the last time window of the morning (8 a.m. to 9 a.m.), the buses start their trips from middle schools to serve elementary students. Finally, after serving all three levels of schools, the buses make deadhead trips to the depot.

In the afternoon time window, the buses are running with a reverse process from the morning trips. The buses start with deadhead trips from the depot to high schools to pick up high school students, then, after delivering students to their stops or homes, the buses go directly to middle schools to pick up and deliver middle school students. After delivering the last student, the buses head to elementary schools to pick up and deliver elementary students. After delivering all elementary students, the buses make deadhead trips to the depot. Figure 6 and Figure 7 show the simplified conceptual operations of school buses during the morning and afternoon time windows.

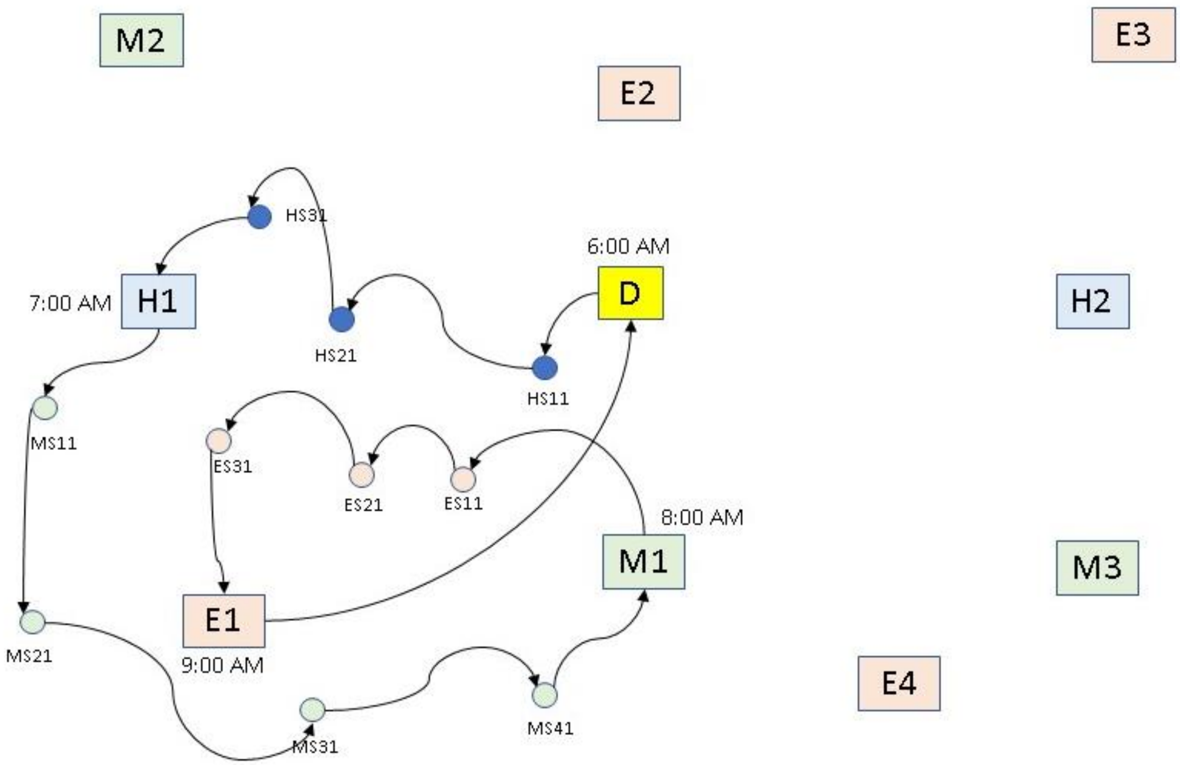

Figure 6: Simplified conceptual operation of school buses in a.m. trips

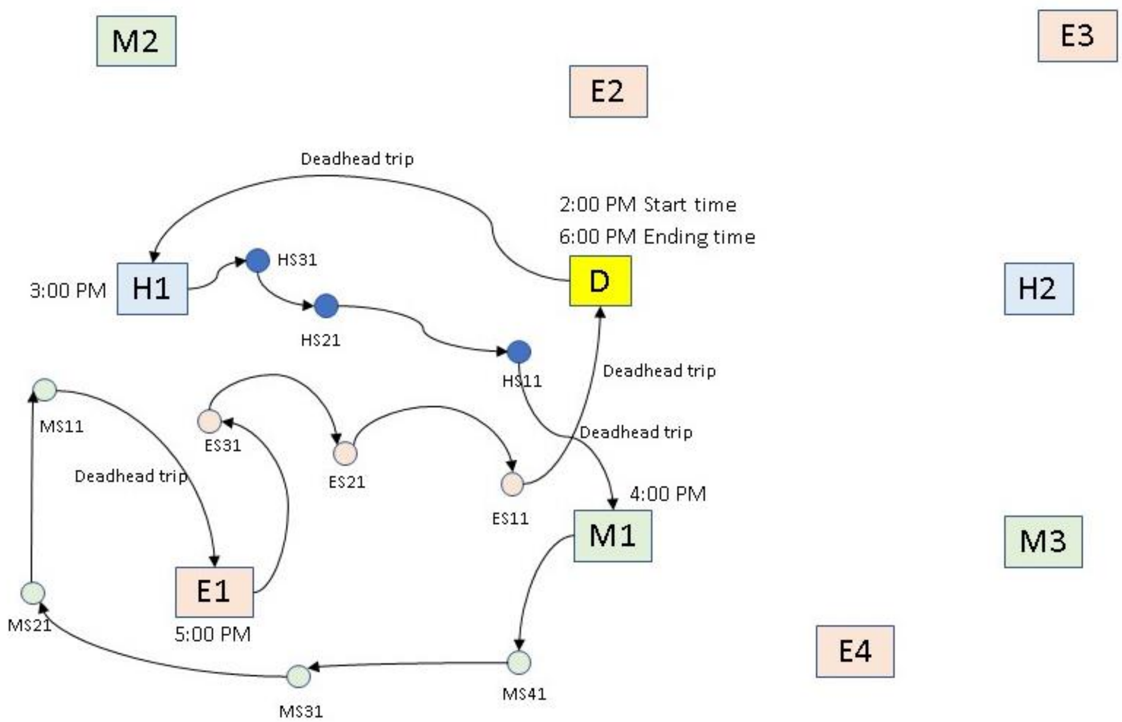

Figure 7: Simplified conceptual operation of school buses in p.m. trips

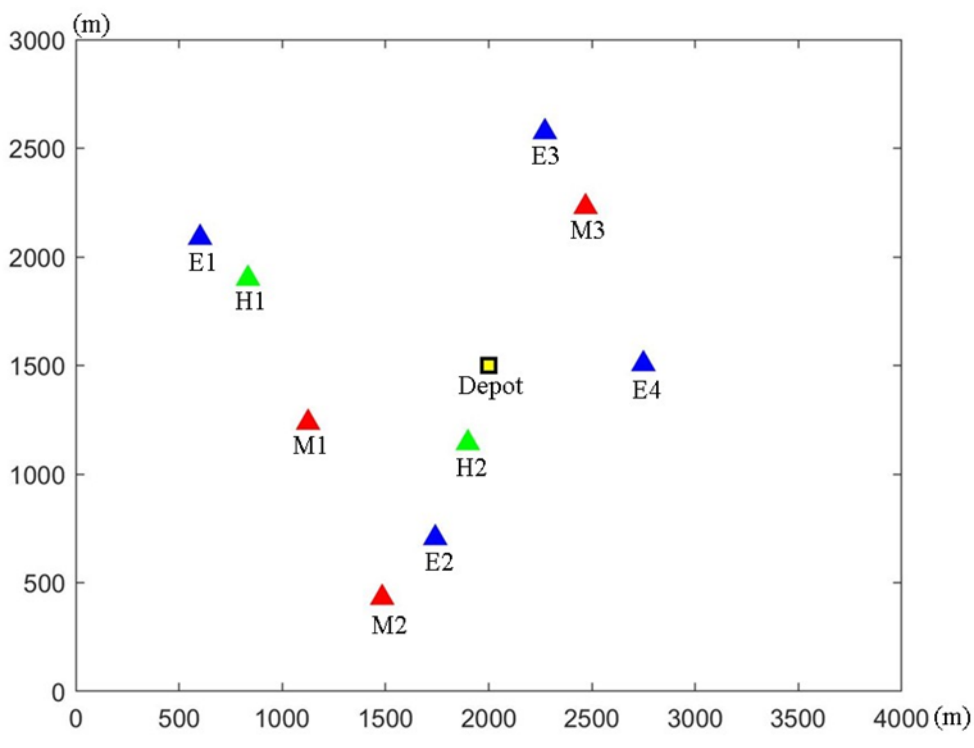

Figure 8: Locations of the schools in the hypothetical network

5.3 Results

The algorithm successfully generated the school bus routings for both the morning and afternoon periods. Figure 9 and Figure 10 show the routings for the two high schools in the morning period. Figure 11–Figure 13 show the routings for three middle schools in the morning. Figure 14–Figure 17 show the routings for the four elementary schools in the morning period. Figure 18–Figure 26 also show the routings in the afternoon period for the two high schools, three middle schools, and four elementary schools, respectively. All routings were constrained by DOC 3, which means that students' travel time in the vehicle should be less than three times their direct travel time from their home to the school or school to home.

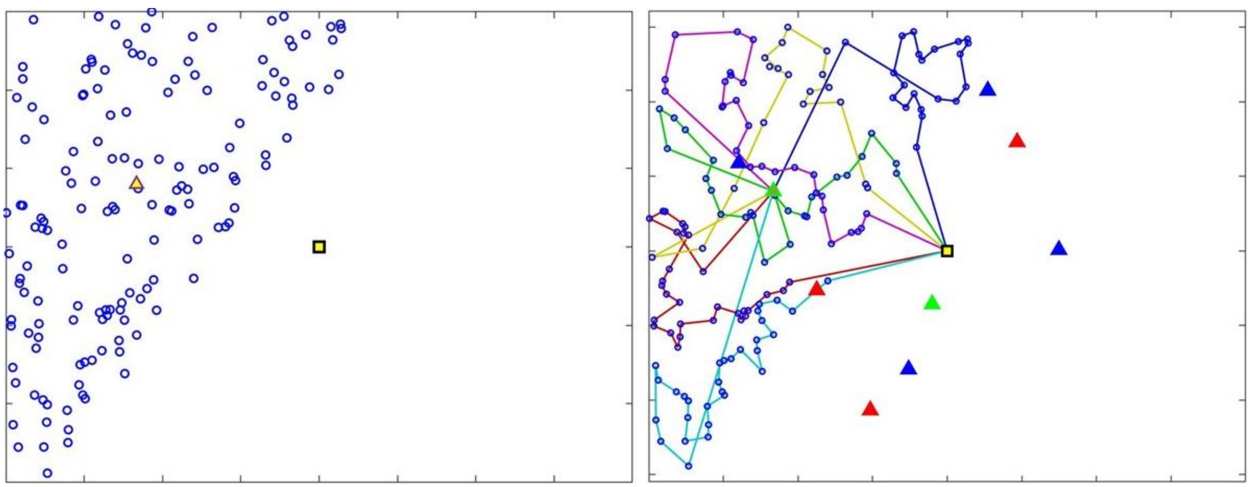

Figure 9: High school H1 student distribution and routings (a.m., DOC = 3)

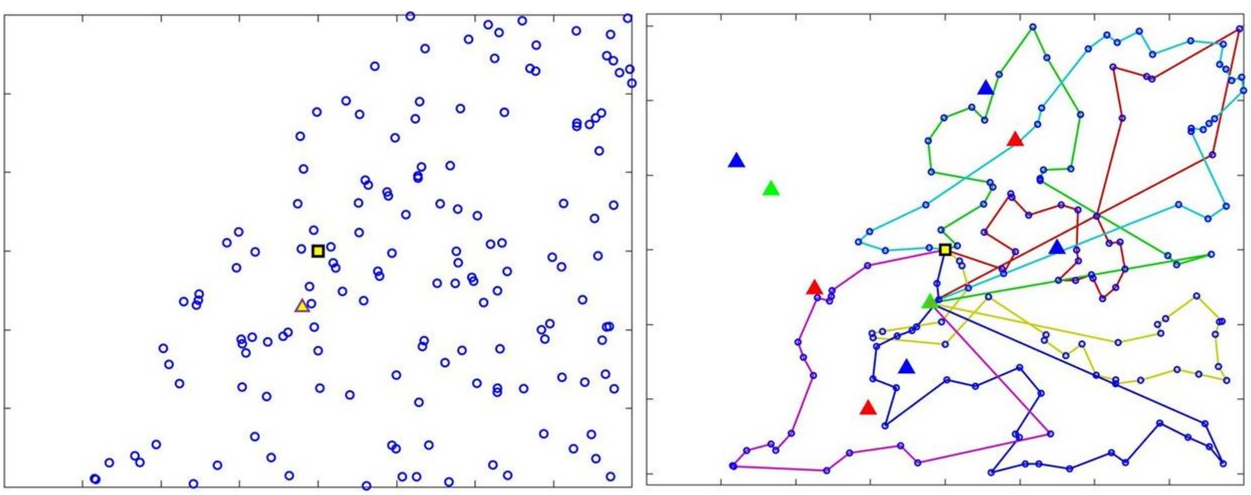

Figure 10: High school H2 student distribution and routings (a.m., DOC = 3)

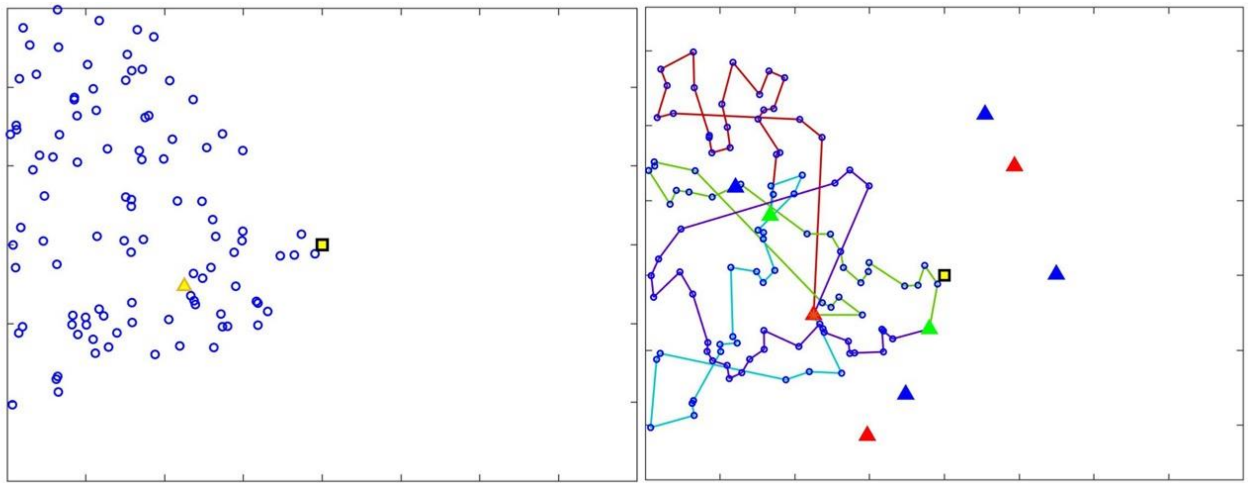

Figure 11: Middle school M1 student distribution and routings (a.m., DOC = 3)

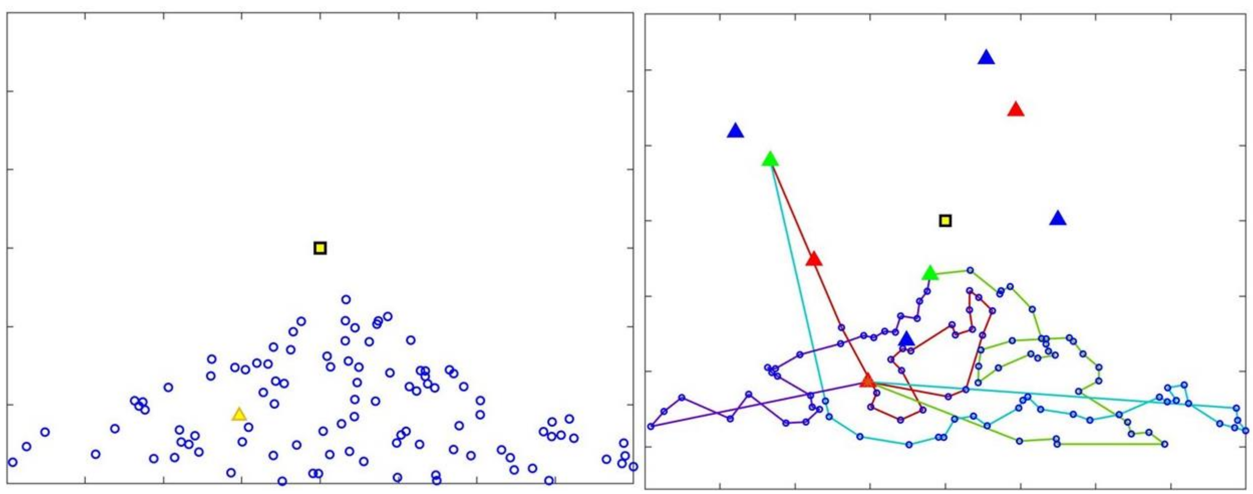

Figure 12: Middle school M2 student distribution and routings (a.m., DOC = 3)

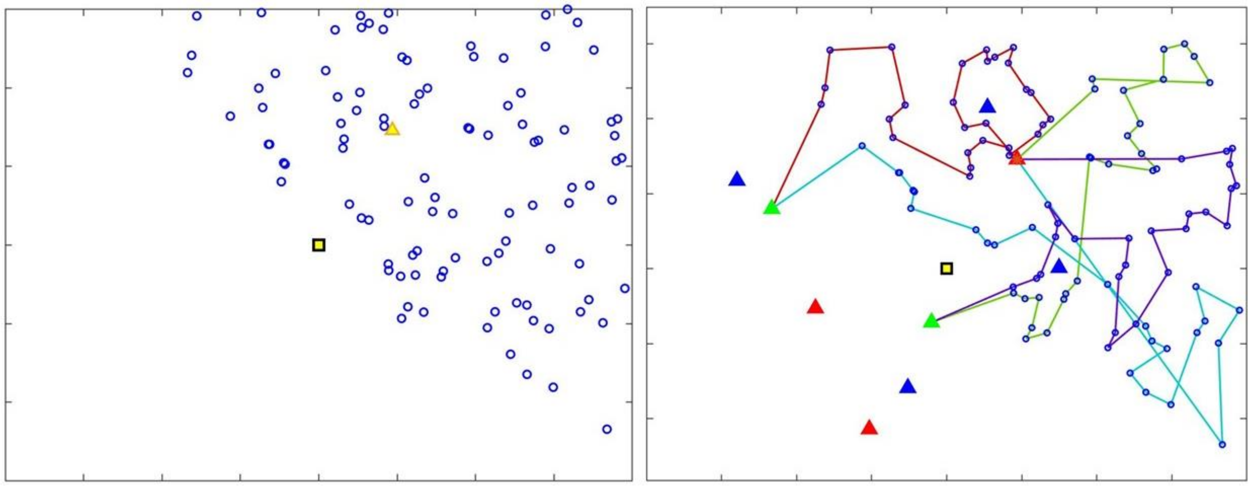

Figure 13: Middle school M3 student distribution and routings (a.m., DOC = 3)

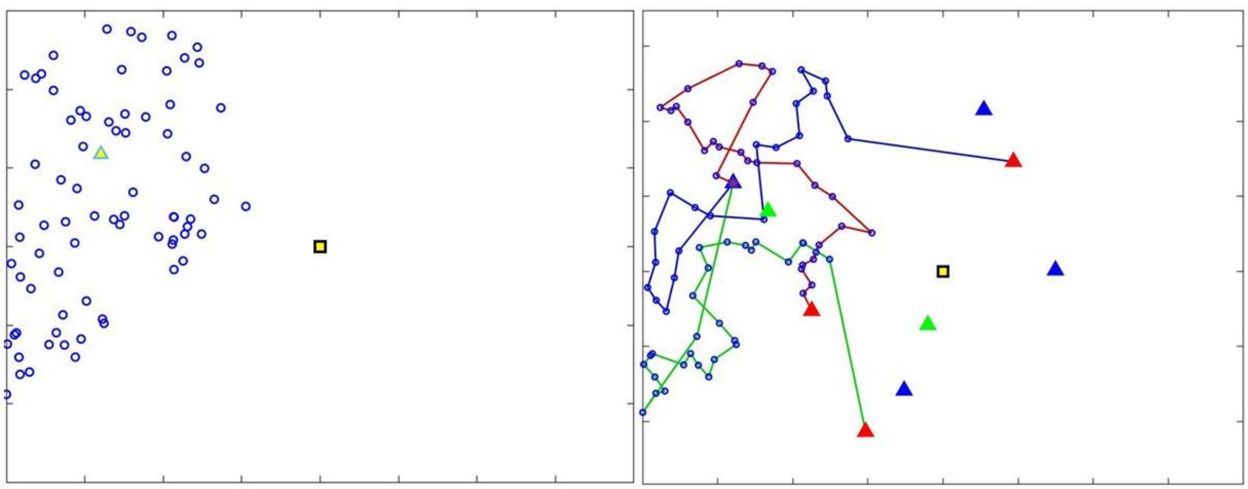

Figure 14: Elementary school (E1) student distribution and routings (a.m., DOC = 3)

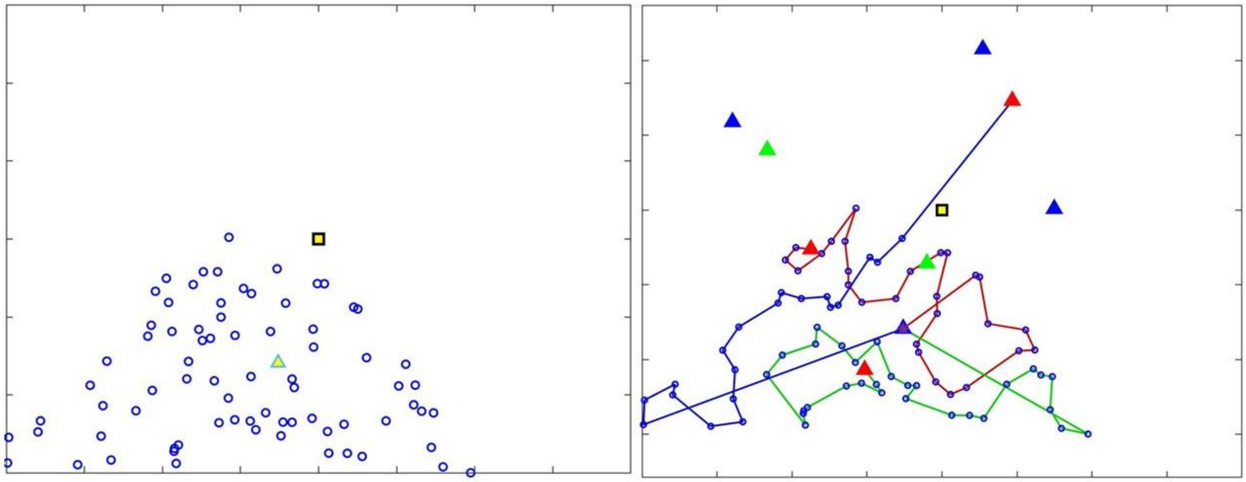

Figure 15: Elementary school E2 student distribution and routings (a.m., DOC = 3)

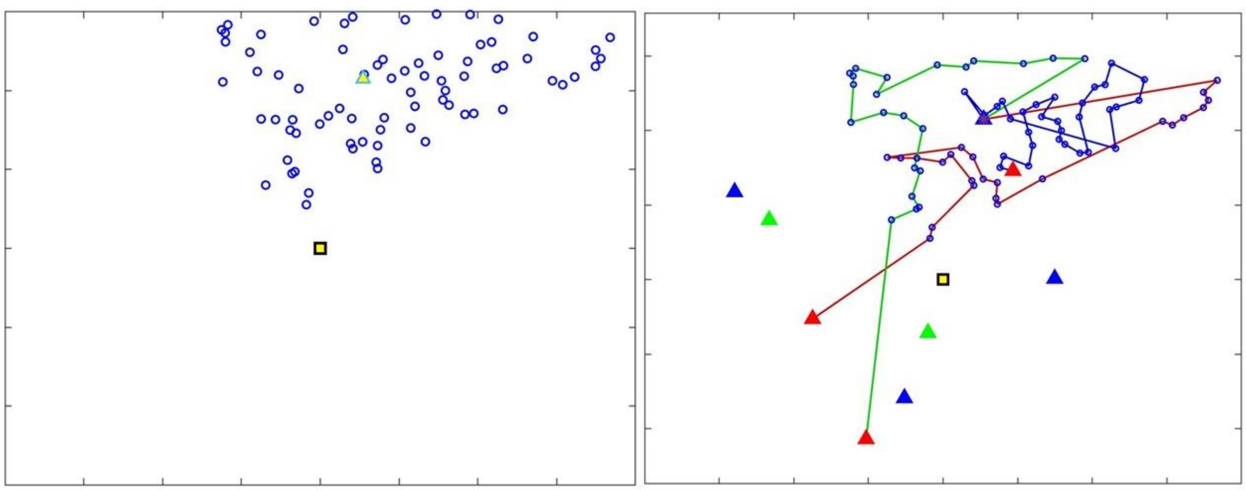

Figure 16: Elementary school E3 student distribution and routings (a.m., DOC = 3)

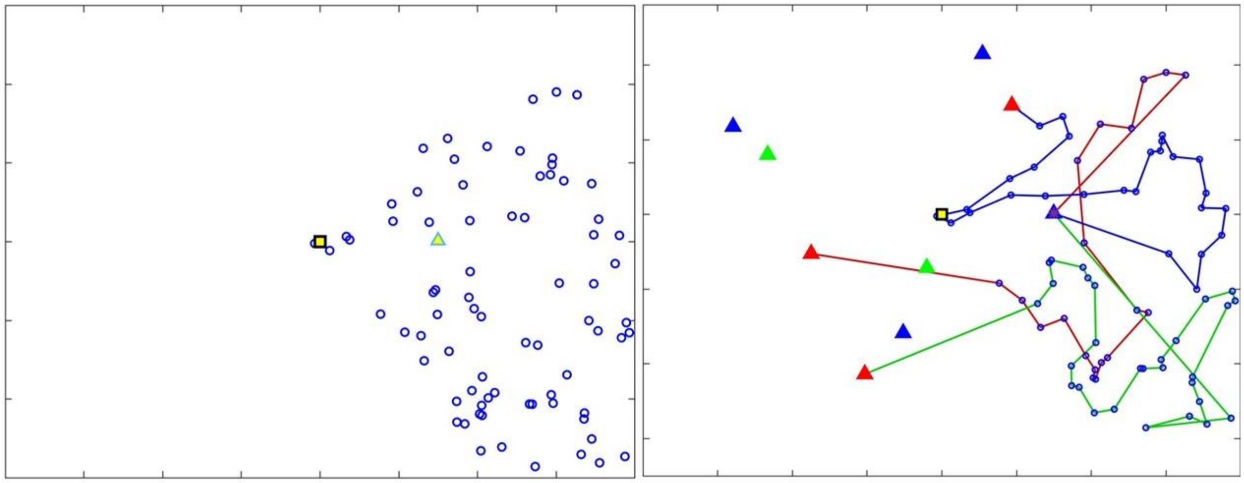

Figure 17: Elementary school E4 student distribution and routings (a.m., DOC = 3)

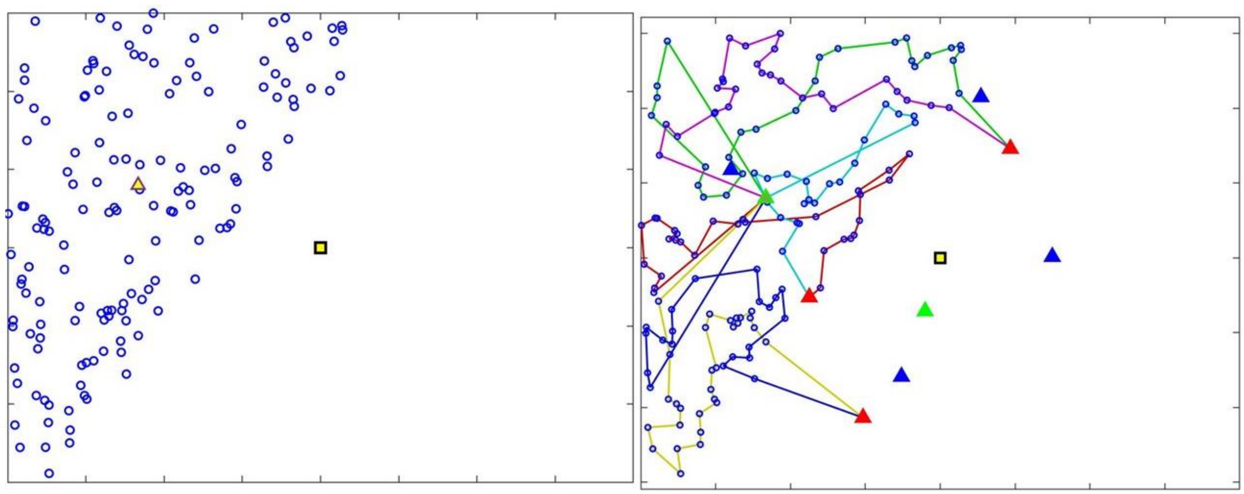

Figure 18: High school H1 student distribution and routings (p.m., DOC = 3)

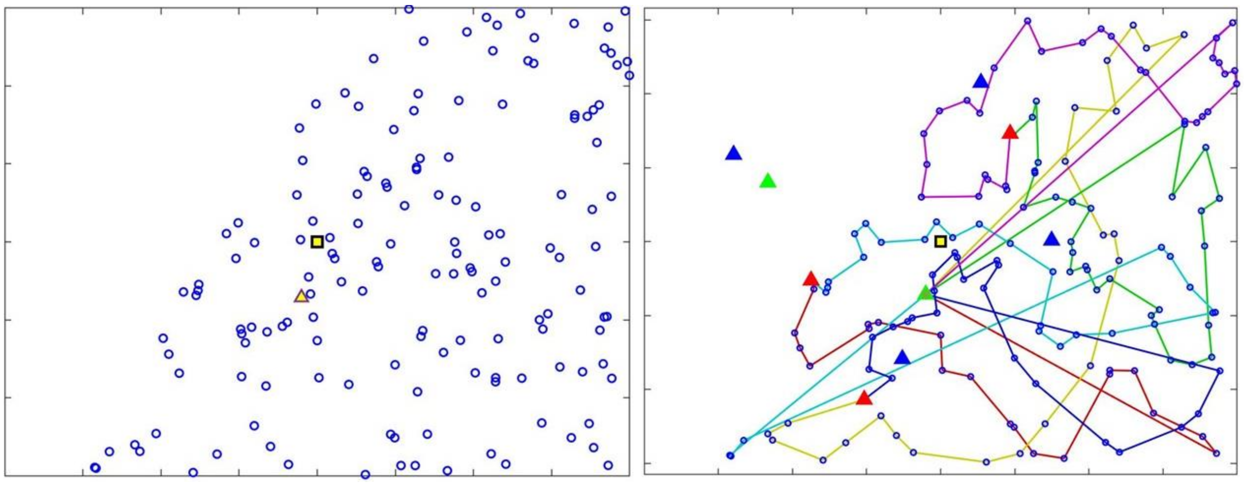

Figure 19: High school H2 student distribution and routings (p.m., DOC = 3)

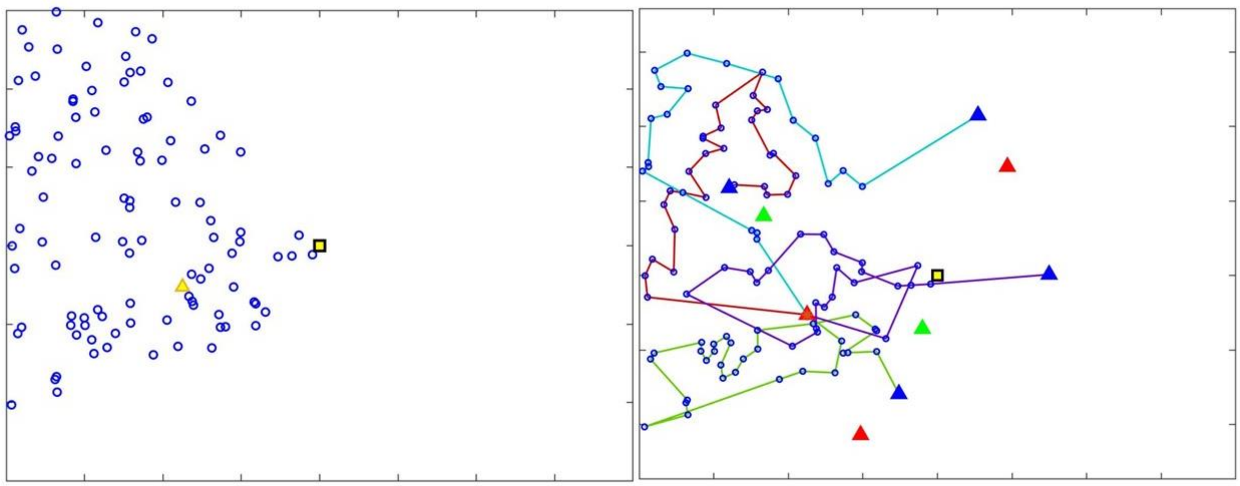

Figure 20: Middle school M1 student distribution and routings (p.m., DOC = 3)

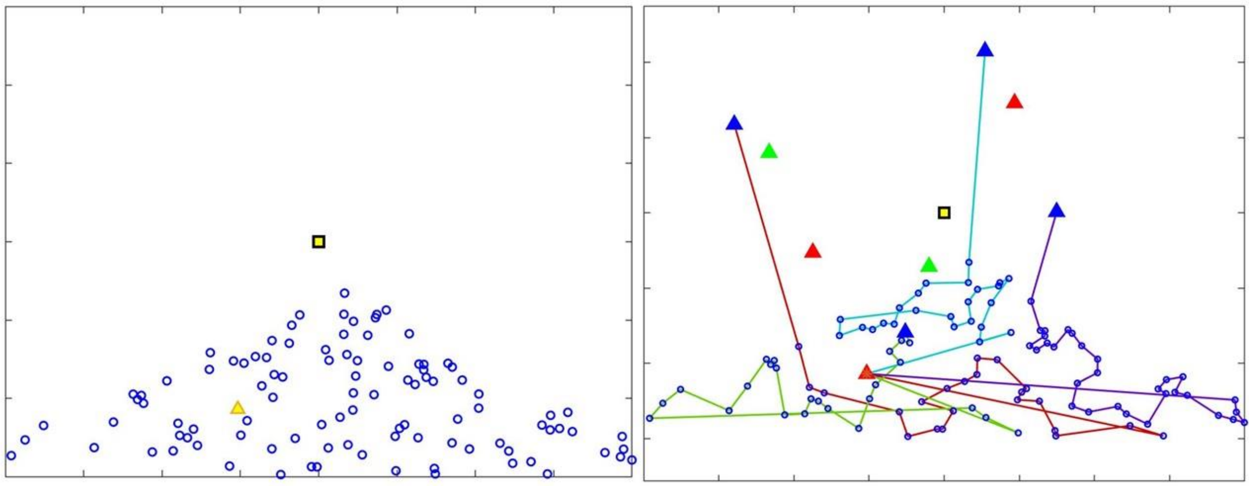

Figure 21: Middle school M2 student distribution and routings (p.m., DOC = 3)

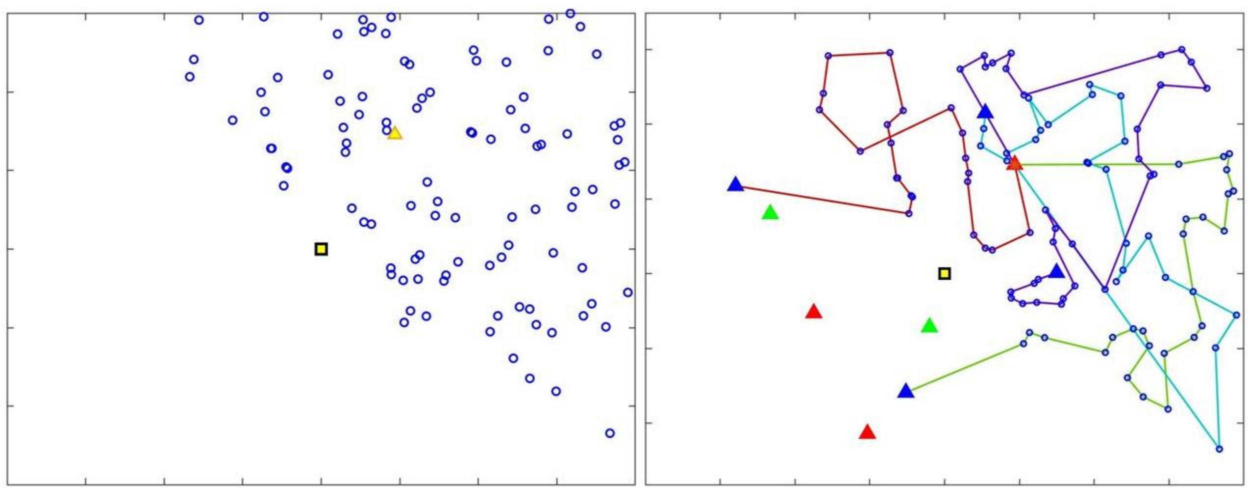

Figure 22: Middle school M3 student distribution and routings (p.m., DOC = 3)

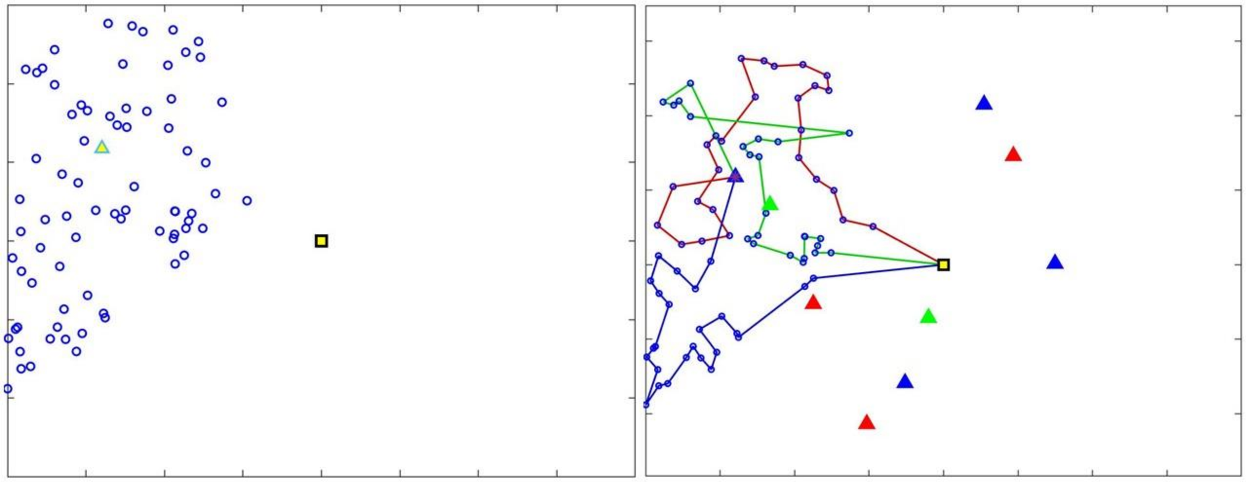

Figure 23: Elementary school E1 student distribution and routings (p.m., DOC = 3)

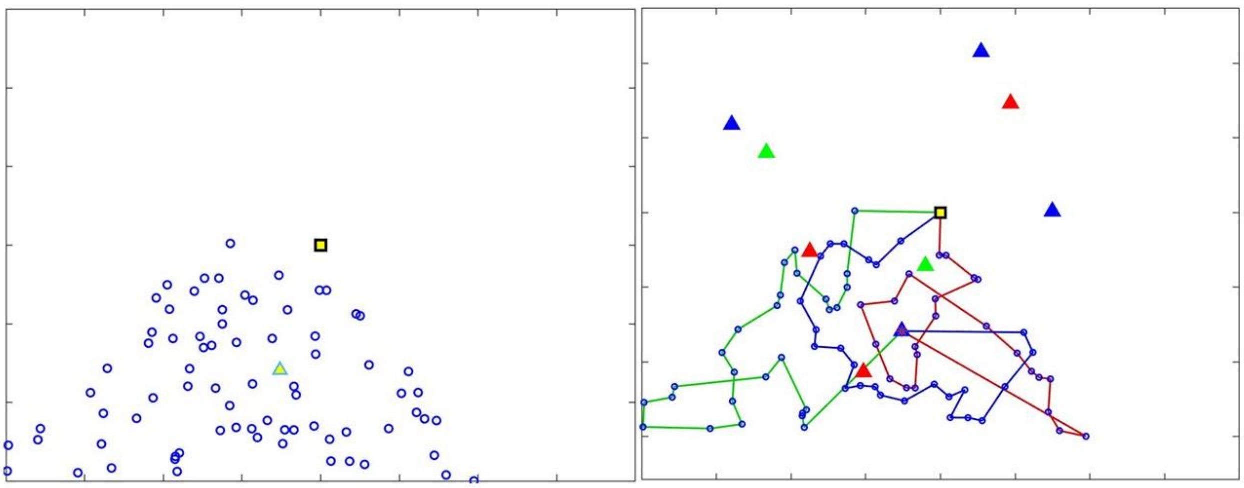

Figure 24: Elementary school E2 student distribution and routings (p.m., DOC = 3)

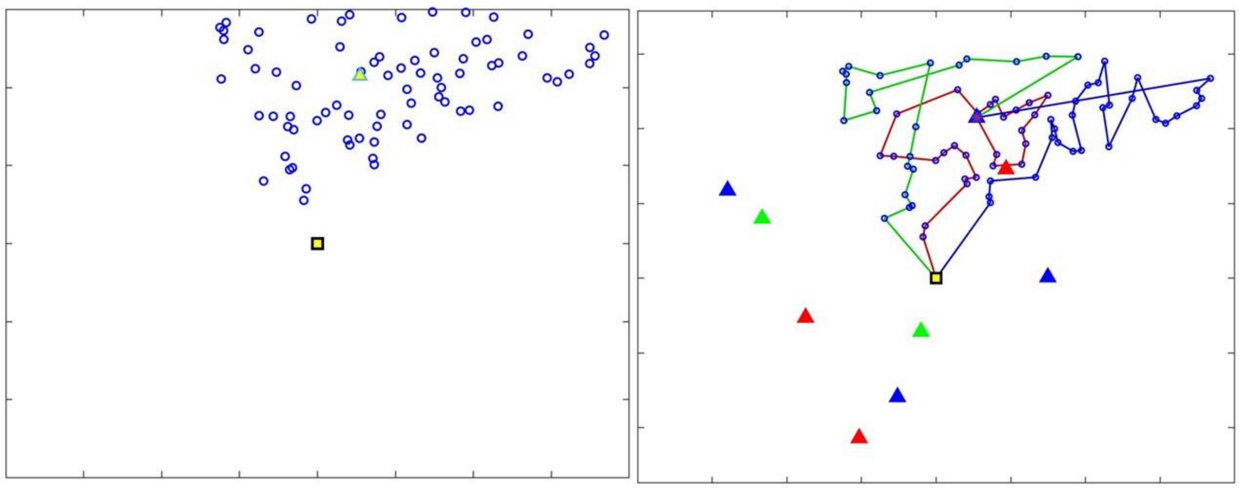

Figure 25: Elementary school E3 student distribution and routings (p.m., DOC = 3)

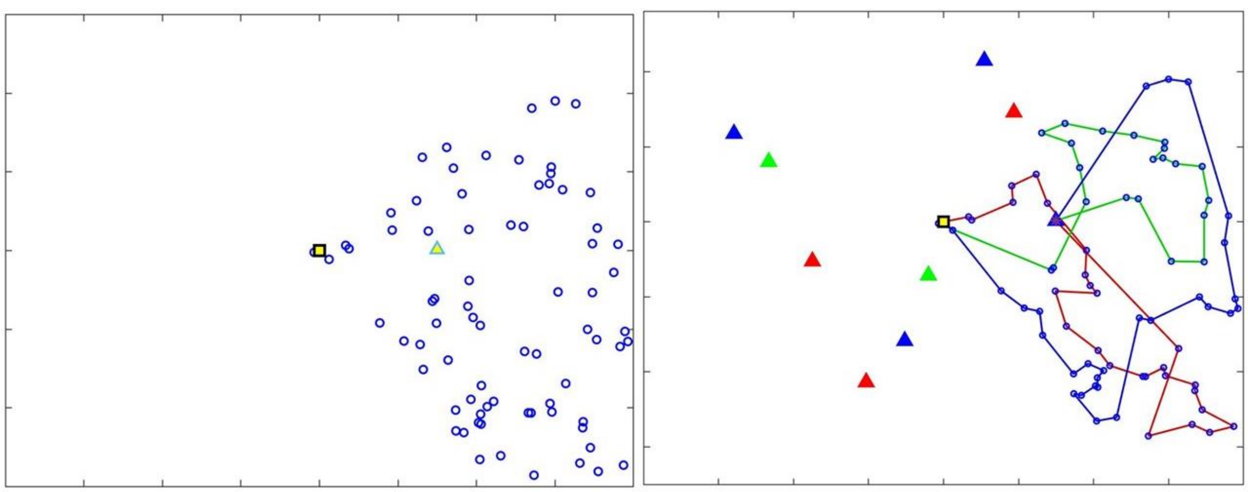

Figure 26: Elementary school E4 student distribution and routings (p.m., DOC = 3)

The authors conducted other routings without any constraints for the directness of student trip (i.e., no DOC constraint in the algorithm) as well to compare the results with the routings with the maximum DOC of 3. Table 2 shows the comparison between two time periods (morning and afternoon), two DOC types (DOC = 3 and no DOC) and three levels of school (high school, middle school, and elementary school).

As shown in the table, the cost components for morning and afternoons are different. Indeed, as shown in the above figures, the routings for morning and afternoons are different because they have different origins and destinations. The maximum DOC affects routings and the costs slightly. As expected, without the maximum DOC constraint, the results lean toward minimizing the total costs since the constraint was relaxed. The average costs for elementary schools were the lowest. The costs for high schools were the highest since high schools draw from wider areas and have more students, whereas elementary schools cover narrower areas with fewer students.

Table 3 compares total costs and number of students that experience travel times more than DOC 3. Without DOC constraints, total costs are less than the routings with the constraint of the maximum DOC. However, as shown in the table, without the maximum DOC constraint, there were students who spent much more time in the vehicle with more than DOC 3 while no students had more than DOC 3 trips when routings were developed with the maximum DOC constraint. Therefore, a decision must be made as to whether the school bus routings should be developed without the maximum DOC constraint, which means that the in-vehicle travel time of some students will be more than three times their shortest direct travel time, or whether the school bus routings should include a maximum DOC constraint to avoid any long trips for students, although the total costs will be slightly higher.

Although the high schools, middle schools, and elementary schools serve the same number of students (260 students) with the same number of total buses (12 buses), the total cost for high schools is more than that of middle schools, and the total cost for middle schools is more than that of elementary schools. High schools serve the widest area with more dispersed students than do middle schools, and middle schools serve wider areas and more dispersed students than elementary schools do.

Table 2: Comparison of Models with DOC 3 and No DOC

DOC Ratio	School	Time Window	Average Total Student travel time (hr)	Average Total Bus Travel Distance (km)	Average Total Student Travel Cost ($)	Average Total Bus Operating Cost ($)	Average Total Cost ($)
3	High Schools	Morning	12.59	4.10	125.87	12.30	138.17
3	Middle Schools	Morning	8.34	2.61	83.35	7.82	91.17
3	Elementary Schools	Morning	6.10	1.91	60.95	5.74	66.69
NO DOC	High Schools	Morning	12.67	3.55	126.66	10.64	137.30
NO DOC	Middle Schools	Morning	8.44	2.10	84.38	6.30	90.68
NO DOC	Elementary Schools	Morning	6.17	1.61	61.67	4.84	66.51
3	High Schools	Afternoon	12.19	3.92	121.85	11.76	133.62
3	Middle Schools	Afternoon	8.08	2.53	80.76	7.59	88.35
3	Elementary Schools	Afternoon	6.05	1.73	60.47	5.18	65.65
NO DOC	High Schools	Afternoon	12.43	2.91	124.25	8.73	132.98
NO DOC	Middle Schools	Afternoon	8.22	2.03	82.22	6.10	88.32
NO DOC	Elementary Schools	Afternoon	6.11	1.42	61.09	4.26	65.36

Table 3: Comparison of Total Costs and Indirect Trips with DOC 3 and No DOC

Time Window	School	Total Costs		Number of Students with Higher than DOC 3	
		DOC 3	No DOC	DOC 3	No DOC
Morning	High Schools	276.33	274.60	0	5
	Middle Schools	273.52	272.04	0	46
	Elementary Schools	266.76	266.03	0	22
Afternoon	High Schools	267.23	265.97	0	3
	Middle Schools	265.06	264.96	0	46
	Elementary Schools	262.60	261.42	0	26

6 Summary and Conclusions

In the first part of this report, we outlined the challenges faced by the BPS bus logistics team. We stressed the various issues of the collected GPS data and their impact on the performance of Versatrans, the tool used by BPS to design bus routes. Obtaining relevant travel time and velocity data on each link of the network is crucial in order to enable Versatrans to generate relevant routes, avoid congested regions, and possibly eliminate some of the expected congestion that is caused by BPS buses.

In the second part, a more realistic school bus routing algorithm which serves three different levels of schools in a single framework was developed. In most counties or cities, the public schools cover all three levels—high school, middle school, and elementary school—and the school buses transport the three levels of students during three different time windows. In the morning, the school buses depart from the depot, first collecting and then delivering high school students. The buses depart from the high school to pick up middle school students, and after they are taken to school, the buses gather elementary school students and ferry them to school before returning, empty, to the depot. In the afternoon, the buses depart from the depot and go to high schools to pick up students. They deliver the high school students to their homes, head to middle schools next, and then go to the elementary schools until all students are dropped off. Once empty, the buses return to the depot.

This research makes two main contributions. The first one is to consider three levels of schools in three separate time windows in a single framework to optimize the entire routing. Second, the algorithm considers the maximum DOC for all individual students, which enforces all student trips to be within a certain travel circuity. Because lengthy travel for certain students is one of the major complaints about school bus routing, it is believed that including the maximum DOC as a constraint of the algorithm can improve the level of service for some students.

As a result of the algorithm, the school bus routings for both the morning and afternoon periods were successfully generated. The authors conducted additional routings with and without any constraints for the directness of student travel. The DOC affects routings and increases the costs slightly. However, no student has travel time on the bus more than the selected DOC of 3; in contrast to 73 students experiencing longer travels without the DOC constraint. Another observation is the larger the area served the higher the cost. That is why, the cost for high schools is superior to that of middle schools which is in turn superior to elementary schools. Changing the origin and destinations affect the routing: The cost components for morning and afternoons are different, and, indeed, the routings for morning and afternoons are different.

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