



Final Report

Optimal Automated Demand Responsive Feeder Transit Operation and Its Impact

Principal Investigator
Young-Jae Lee, Ph.D.

Associate Professor, Department of Transportation and Urban
Infrastructure Studies, Morgan State University, 1700 E. Cold Spring Lane,
Baltimore, MD 21251

Tel: 443-885-1872; Fax: 443-885-8218; Email: YoungJae.Lee@morgan.edu

Co-Principal Investigator
Amirreza Nickkar

Graduate Research Assistant, Department of Transportation and Urban
Infrastructure Studies, Morgan State University, 1700 E. Cold Spring Lane,
Baltimore, MD 21251

Email: amnic1@morgan.edu

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Prepared for the Urban Mobility & Equity Center, Morgan State University, CBEIS 327, 1700 E. Coldspring Lane,
Baltimore, MD 21251



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16. Abstract <p>Although demand responsive feeder bus operation is possible with human-driven vehicles, it has not been very popular and mostly available as a special service because of the high operating costs due to the intensive labor costs as well as advanced real-time information technology and complicated operation. However, once automated vehicles become available, small-sized flexible door-to-door feeder bus operation will become more realistic, thanks to recent technological advances and business innovations by the transportation network companies (TNCs). So, preparing for the automated flexible feeder service is necessary to catch the rapid improvement of automated vehicle technology.</p> <p>Therefore, this research developed an algorithm for the optimal flexible feeder bus routing, which considers relocation of buses for multi-stations and multi-trains, using a simulated annealing (SA) algorithm for future automated vehicle operation. An example was developed and tested to demonstrate the developed algorithm. The algorithm successfully handled relocating the buses when the optimal bus routings were not feasible with the available buses at certain stations. Furthermore, the developed algorithm limited the maximum Degree of Circuitry for each passenger while minimizing total cost, including total vehicle operating costs and total passenger in-vehicle travel time costs. Unlike fixed route mass transit, small vehicle demand responsive service uses flexible routing, which means lower unit operating costs not only decrease total operating costs and total costs but also can affect routing and impact network characteristics. In the second part of this research, optimal flexible demand responsive feeder transit networks were generated with various unit transit operating costs using the developed routing optimization algorithm. Then network characteristics of those feeder networks were examined and compared.</p> <p>The results showed that when unit operating costs decline, total operating costs and total costs obviously decline. Furthermore, when unit operating costs decline, the average passenger travel distance and total passenger travel costs decline while the ratio of total operating costs per unit operating costs increases. That means if unit operating costs decrease, the portion of passenger travel costs in total costs increases, and the optimization process tends to reduce passenger costs more while reducing total costs. Assuming that automation of the vehicles reduces the operating costs, it will reduce total operating costs, total costs and total passenger travel costs as well.</p>			
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ABSTRACT

Although demand responsive feeder bus operation is possible with human-driven vehicles, it has not been very popular and mostly available as a special service because of the high operating costs due to the intensive labor costs as well as advanced real-time information technology and complicated operation. However, once automated vehicles become available, small-sized flexible door-to-door feeder bus operation will become more realistic, thanks to recent technological advances and business innovations by the transportation network companies (TNCs). So, preparing for the automated flexible feeder service is necessary to catch the rapid improvement of automated vehicle technology.

Therefore, this research developed an algorithm for the optimal flexible feeder bus routing, which considers relocation of buses for multi-stations and multi-trains, using a simulated annealing (SA) algorithm for future automated vehicle operation. An example was developed and tested to demonstrate the developed algorithm. The algorithm successfully handled relocating the buses when the optimal bus routings were not feasible with the available buses at certain stations. Furthermore, the developed algorithm limited the maximum Degree of Circuity for each passenger while minimizing total cost, including total vehicle operating costs and total passenger in-vehicle travel time costs.

In order to evaluate the impact of the acceptable individual passengers' maximum travel time, four types of maximum Degree of Circuitries (2.5, 3, 3.5, and 4) were applied. As expected, with the higher individual passengers' maximum Degree of Circuity, the minimum number of buses used, vehicle traveled distance and total costs of the service decreased due to the more relaxed constraint, although the lower individual passengers' maximum Degree of Circuity ratio guarantees lower maximum circuitry of travel to all users.

Unlike fixed route mass transit, small vehicle demand responsive service uses flexible routing, which means lower unit operating costs not only decrease total operating costs and total costs but also can affect routing and impact network characteristics. In the second part of this research, optimal flexible demand responsive feeder transit networks were generated with various unit transit operating costs using the

developed routing optimization algorithm. Then network characteristics of those feeder networks were examined and compared.

The results showed that when unit operating costs decline, total operating costs and total costs obviously decline. Furthermore, when unit operating costs decline, the average passenger travel distance and total passenger travel costs decline while the ratio of total operating costs per unit operating costs increases. That means if unit operating costs decrease, the portion of passenger travel costs in total costs increases, and the optimization process tends to reduce passenger costs more while reducing total costs. Assuming that automation of the vehicles reduces the operating costs, it will reduce total operating costs, total costs and total passenger travel costs as well.

Key words: *Automated Transit, Demand Responsive Transit, Feeder Bus, Vehicle Routing Problem, Optimization*

1. INTRODUCTION

In the past, automated transit was regarded as Personal Rapid Transit or Automated Rail Transit. Now, people are talking about autonomous vehicle technology, which for public transit means autonomous bus transit, ridesharing and carsharing. Transportation network companies (TNCs), such as Uber, and many car manufacturers, including General Motors, reported recently that fully automated vehicles will be available by 2020-2030 (Fazlollahtabar & Saidi-Mehrabad, 2015). This means that carsharing and ridesharing using autonomous vehicles will become a reality within a decade. While car manufacturing and technology companies are developing and improving technologies for automated vehicles, it seems that policy, planning and other essential elements for the effective implementation of automated vehicles for transit service still lag behind the technology (Krueger, Rashidi, & Rose, 2016).

Autonomous and connected vehicles (CVs) will change the paradigm for transportation users and industries, as well as for public transportation (including mass transit, ridesharing and carsharing) (Shin, Callow, Dadvar, Lee & Farkas, 2015; Shin, Callow, Farkas & Lee, 2016). Not only can they improve travel safety (e.g., reducing crashes), but they also will change our lives and travel patterns. When autonomous vehicles are available, users' travel behaviors and modal choices will become completely different. Autonomous vehicles likely will result in reductions in car ownership and increases in carsharing and ridesharing (Bizon, Dascalescu, & Tabatabaei, 2014). Also, small automated transit vehicles could be utilized to pick up passengers as a feeder for mass transit – such as bus, LRT, and metro – because of lower operational costs due to no labor costs.

Although demand responsive feeder bus operation is possible even with human-driven vehicles, it is not very popular and mostly available as a special service because of the high operating costs due to the intensive labor costs as well as advanced real-time information technology and complicated operation. However, once automated vehicles become available, small-sized flexible door-to-door feeder bus operation will become more realistic, thanks to recent technological advances and business innovations by the transportation network companies (TNCs). So, preparing for the automated flexible feeder service is necessary to catch the rapid improvement of automated vehicle technology.

Before procuring and operating automated transit vehicles, it is extremely important to determine what future transit customers want and expect from them. For example, whether transit customers prefer a traditional type of transit service (e.g., fixed-route) with an automated small-sized feeder transit or a more flexible service, similar to that provided by TNCs, transit agencies should prepare their future transit service accordingly.

This research developed the algorithm for the optimal flexible operation of small-sized automated feeder transit vehicles as the first step in predicting future transit users' mode choice and travel behavior. More specifically, this algorithm considers multi-station and multi-train situations while feeder buses are being relocated, if necessary. Then, the second part of this research generated optimal flexible demand responsive feeder transit networks with various unit transit operating costs using the developed routing optimization algorithm and compared the network characteristics of those feeder networks in order to examine the impact of the automated feeder transit operation.

This study serves as the basis for evaluating the efficiency of the automated feeder service, which then can be compared to automated ridesharing and carsharing services in the future. Eventually, these studies will help predict users' travel behaviors and modal choices between the automated ridesharing/carsharing operation and the automated feeder service for mass transit.

2. LITERATURE REVIEW

The vehicle routing problem (VRP) has been a very popular topic in the research field in recent decades. Many optimal vehicle routing algorithms have been developed, such as the traveling salesman problem, transit routing, delivery and pickup routing, multi-vehicle routing, and flexible ridesharing routing. Among the many types of VRP, most relevant categories were reviewed in this section.

2.1. Feeder Bus Service

Feeder bus services impact commuters' travel choice decisions and consequently the proportion of each mode used daily. Increasing the rate of passenger demand for the transit system could be one of the impacts; therefore enhancing social equality and welfare for the community and boosting operator revenue are expected results (Li, Lam, & Wong, 2009). In recent years, considering the rapid development of public transit, numerous studies have begun to focus more on feeder-bus network optimization. Most of the studies in recent years were focused on developing a feeder-bus network.

Generally, two approaches are used to model feeder-bus route designs. One is solving the feeder-bus network design problem (FBNDP) and the second one is a heuristic feeder route generation algorithm (HFRGA). For the first approach, Kuah & Perl (1988) are the pioneers of FBNDP. They provided a mathematical formulation for the many-to-one network design problem and solved the model by using a heuristic method based on the saving approach. A large number of studies have been conducted to develop and extend FBNDP. Martins & Pato (1998) used a combined heuristics method based on sequential savings and Tabu search algorithms to solve the NP-Hard FBNDP. Kuan, Ong, & Ng (2004) and Kuan, Ong, & Ng (2006) in two different studies tried to solve FBNDP using implemented Simulated Annealing (SA), Tabu Search (TS), Ant Colony (ACO), and Genetic Algorithms (GAs). They believed that SA could be a good algorithm when the issue is finding a better and more sophisticated neighborhood structure to reach high-quality results.

Shariat Mohaymany and Gholami in two different studies (2010 & 2011) implemented a modified ACO algorithm to solve FBNDP which could construct routes and choose terminals simultaneously. The results of these studies show multimodal networks are more effective at decreasing user costs than unimodal networks, and using smaller buses like minibuses rather than conventional buses will reduce operating costs. Ciaffi, Cipriani, & Petrelli (2012) solved FBNDP using a set of heuristics. They divided the procedure for solving the problem into two phases. In the first phase, the procedure generated feasible routes based on the traveling salesman problem (TSP) and K-shortest path algorithms for improving the intermodal interaction, and in the second phase, the model found an optimal or near-

optimal network of routes with the associated frequencies based on the GA. Recently, Lin & Wong (2014) proposed a multi-objective model to locate feeder-bus routes for a specific metro station in Taiwan as the case study taken by minimizing route length, and maximum route travel time, and maximizing service coverage for trip generation.

For the second approach, Shrivastav & Dhingra (2001) introduced a new approach to feeder-bus route design by implementing HFRGA; thereafter, Shrivastava & O'Mahony (2006, 2007 & 2009) extend this algorithm by using GAs and a hybrid algorithm. They control the lengths of feeder routes and balance user need and operator requirements in the proposed algorithms.

Although most of the studies on the optimization of feeder-bus networks considered a network approach, which involved a feeder-bus network that maximized joining the feeder-bus routes with a given rail public transit, some studies used an analytic approach to optimize the feeder-bus network (Almasi, Mirzapour Mounes, Koting, & Karim, 2014). In this approach, the problem maximizes the coverage of passengers' access to all of the stops one by one. Regarding this approach, for the first time, Kuah & Perl (1988) introduced a model for designing an optimal feeder bus network considering bus route locations, bus headways, and stop spacing. The objective of this model was minimizing total user and operator costs. They pointed out that a combination of these three variables at the same time as a function of system parameters and demand density can provide appropriate results. Chien & Yang (2000) implemented the exhaustive search (ES) algorithm as a metaheuristic method based on a many-to-many travel pattern to optimize feeder bus route location and operating headway. The results of this work showed that the user in-vehicle and access costs dominate the optimal route location. Chien, Yang, & Hou (2001) extended this study implementing GA to solve the model. The results of the comparison between two algorithms show that although there was no significant difference between the ultimate solutions, GA can solve the problem much faster. Deng, Gao, Zhou, & Lai (2013) used GA to determine optimal feeder-bus operating frequency. They believed that integrating public transit modes in a unit multimodal framework and improving the modes' connectivity is necessary to enhance the competitiveness of public transport.

2.2. Ridesharing

The shared economy can benefit communities. Ridesharing is one of the most efficient elements of the shared economy, one that is able to reduce unnecessary energy and costs and consequently open a way for communities to reach sustainable development in their urban areas (Samól, 2017). There has been a long discussion about coordination methods of ridesharing. Recently, Furuhata, et al. (2013) classified ridesharing into six classes: dynamic real-time ridesharing, carpooling, long-distance ride-match, one-shot ride-match, bulletin-board, and flexible carpooling.

The carpooling problem has some specific features; generally, it is an arrangement of one-time trips rather than allocating a pickup visit for each passenger and instantly matching riders via a complex database system (Kaan & Olinick, 2013). Yan & Chen (2011) proposed an algorithm to solve the carpooling problem based on Lagrangian relaxation and a heuristic for the upper bound solution. They pointed out as a result, the model may increase the vehicle usage and save the passengers time. Some studies worked on fleet sizing of the carpooling problem. Karamanis, Niknejad, & Angeloudis (2017) provided a mixed-integer programming (MIP) algorithm for helping with the fleet size, depot location, and the number of Shared Autonomous Vehicles (SAV) problem. They concluded MIP could be an appropriate algorithm for SAV fleet establishment, especially in a larger network.

Developing the vanpooling assignment problem is the other problem that has been considered in recent years. This problem generally pursues two goals: decreasing commuting costs of passengers (Concas, Winters, & Wambalaba, 2005; Ungemah, Rivers, & Anderson, 2006) and assessing the level of participation based on incentives and policies (Huang, Yang, & Bell, 2000; Washbrook, Haider, & Jaccard, 2006). Recently, Kaan & Olinick (2013) provided two models, the Minimum Cost Vanpool Assignment Model (MCVAM) and the Two-Stop MCVAM (TSMCVAM) model, to solve the vanpool assignment problem by minimizing total per-trip costs. They used heuristics to solve the problem and the results clearly showed TSMVCAM could bring significant cost savings for users in terms of both time and trip cost.

2.3. Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD)

Min (1989) for the first time proposed VRPSPD – inspired by a distribution problem of a public library – and applied a heuristic method to solve this real-life problem. Then, many studies tried to solve this problem by various methods. Angelelli & Mansini (2002) could solve this problem with an exact algorithm. Other studies tried to solve the problem by heuristic and metaheuristics algorithms. Because of the structure of VRPSPD, very few studies used exact approaches, and the main part of recent studies focused on applying metaheuristics algorithms. GA (Tasan & Gen, 2012), Tabu Search (TS) (Montané & Galvao, 2006), Greedy Particle Swarm Optimization (Ai & Kachitvichyanukul, 2009), Ant Colony (ACO) (Gajpal & Abad, 2009), Iterated Local Search algorithm (Souza, Mine, Silva, Ochi, & Subramanian, 2011), and SA (Wang C. , Mu, Zhao, & Sutherland, 2015) were metaheuristics approaches that were considered in recent years to solve VRPSPD. The most recent studies are summarized and found in Montoya-Torres, J.R. et al.'s work (2015).

2.4. The Pickup and Delivery Problem with Time Windows (PDPTW)

The pickup and delivery problem with time windows (PDPTW) is a generalized version of the vehicle routing problem with time windows (VRPTW) which is focused on finding optimal routing solutions under capacity and time windows constraints (Baldacci, Bartolini, & Mingozzi, 2011). The objective function of the standard PDPTW is minimizing transportation costs while the routes generation is based on serving all demands (Sun, Veelenturf, Hewitt, & Van Woensel, 2018). At first, Dumas, Desrosiers, & Soumis (1991) solved PDPTW by a branch-and-price algorithm and found the exact solution to the problem. After that, by introducing new objectives and constraints, the PDPTW turned out to be an NP-hard problem, and new heuristic and metaheuristic methods have been implemented for solving these problems. Recent studies implemented two-stage metaheuristic approaches by minimizing the number of routes and total travel distance. These approaches include TS (Nanry & Barnes, 2000), SA (Bent & Van Hentenryck, 2006), large neighborhood search (LNS) (Ropke & Pisinger, 2006), and guided ejection search (GES) (Nagata & Kobayashi, 2010).

2.5. The Dial-A-Ride Problem (DARP)

The dial-a-ride problem (DARP) is an extension of the VRPPD for transporting passengers in transit systems with a set of requests for pick-up and delivery from passengers who should be served by a limited number of transit fleet (Cordeau & Laporte, 2003). The objective functions of DARPs generally include minimizing the total routing distance of the fleet, minimizing travel and waiting time of passengers, and maximizing demand (Cordeau & Laporte, 2007). The DARPs belong to the NP-hard problems, and with a condition that the transit system is obligated to schedule vehicles to serve passengers in a determined time range, it turns to the dial-a-ride problem with time window (DARPTW), which is more complex. The structure of DARPTW is very similar to the PDPTW; however, it is more complicated and very sensitive to constraints.

A range of metaheuristics methods to find a near optimal solution of the DARPTW have been implemented in past studies: ACO (Tripathy, Nagavarapu, Azizian, Pandi, & Dauwels, 2017), GA (Cubillos, Urra, & Rodríguez, 2009), SA (Zidi, Zidi, Mesghouni, & Ghedira, 2011), and TS (Belhaiza, 2017). For further details and a comprehensive review of DARP and DARPTW models, the reader is referred to a literature review study by Molenbruch, Braekers, & Caris (2017). Although these studies have provided useful results that can minimize passenger or operator costs, there are still limitations in the implementation of these approaches in rural areas. First, most of these studies tried to consider increased operator revenues by scheduling vehicles on optimal routes even though individual passengers' travel time and traveler preferences are important variables that can change the travel behavior of the traveler. Second, these approaches did not consider relocation of the fleet service despite the fact that in high-demand conditions fleet relocation might be required. Finally, none of studies considered visualization tools to show optimal solutions. Although many studies have investigated the VRSPD, DARPTW, PDPTW and VRPTW problems, very little knowledge is available for solving VRSPD with time window constraints and considering designing the optimal transit network and carsharing, especially including relocation of vehicles. Therefore, in the current study we will provide an algorithm to cover all of these considerations.

3. METHODOLOGY

The authors developed the optimal routing algorithm specifically for the automated demand responsive feeder transit services. The algorithm minimizes total costs, including vehicle operating costs and passenger travel time, while individual passengers' maximum travel times are limited within given maximum travel times. That differentiates this algorithm from the usual delivery-pickup algorithms, which do not consider individual packages' (in this case, passengers') travel times. Moreover, the other innovations of this study are considering relocation of buses and the dynamic nature of the operation, which involves multi-stations and multi-trains. Also, this algorithm applies the Simulated Annealing (SA) (Kirkpatrick, Gelatt, & Vecchi, 1983) algorithm to solve the proposed model.

Generally, the SA algorithm works effectively in permutation-based problems like VRP (Davendra, Zelinka, & Onwubolu, 2010) because this local search algorithm uses random transformations from the neighborhood of all solutions by considering the total costs in VRP problems, which are the total dispatching cost and total travel cost of possible solutions (Hasija & Rajendran, 2004). Among metaheuristics that have provided better solutions, TS and SA are two of the more effective algorithms (especially in the small-scale problems) due to structures of finding an appropriate neighborhood in these algorithms (Bräysy & Gendreau, 2005; Potvin, 2009; Van Breedam, 1996).

3.1. Algorithm

This section presents an efficient algorithm for optimal routing of the autonomous feeder transit services, and the first step of this algorithm is the clustering of passengers; we assumed that all passengers are assigned to certain stations.

At first, the algorithm creates a random series of integers for creating the initial solution (random permutation of integers from 1 to “number of passengers of the related station plus number of feeder buses minus one”). The algorithm allocates feeder buses to passengers depending on the location of greater integers in the generated permutation, and then, based on the order of integers in the generated permutation, the routes of vehicles are determined. For instance, in the presence of two vehicles and 10

passengers, a permutation of integers from 1 to 11 is produced. Suppose that the generated permutation is as follows: Path= [10, 1, 3, 4, 8, 11, 2, 9, 5, 7, 6]. Then the route of the first vehicle would be generated by serving passengers 10, 1, 3, 4, 8, respectively, and the second vehicle route is made by serving passengers 2, 9, 5, 7, 6, respectively. In each iteration, the algorithm tries to improve the solution by searching its neighborhoods. For this purpose, common swap, insertion, and reversion methods were used. The generated solution in the neighborhood of current solution is compared to the current solution, and based on the SA algorithm approach it would be accepted or rejected. The comparison is based on the value of a hypothesized objective function which includes penalties for modeling constraints. The algorithm attempts to reduce the value of the hypothesized objective function. The original and hypothetical objective functions are calculated as follows:

$$Z' = C_o \times \text{Total vehicles travelled distance} + C_T \times \text{Total passengers in vehicle travel time}$$

$$Z = Z' * (1 + 0.5 \times \text{number of passengers not served in time window} + 5 \times \text{max number of passengers in excess of feeder bus capacity})$$

Where C_o is the unit operating cost of each vehicle kilometer and C_T is the time value of each passenger per hour. Finally, the best feasible solution found during the total iterations is presented as the final solution proposed by the algorithm.

If the algorithm fails to find a feasible solution for a certain station, relocations of vehicles would be considered. In this case, the proportion of the number of passengers to the number of feeder buses for the adjacent stations is computed and the algorithm chooses the station with the lower proportion to compensate for the deficiency.

In order to consider individual passengers' acceptable travel times and acceptable circuitry of the routing, Degree of Circuitry (*DOC*) and Maximum Degree of Circuitry (*Max DOC*) as shown in Equations 1 and 2 are introduced in this research (Lee Y.-J. , 2012; Lee, Choi, Yu, & Choi, 2015). The given *Max DOC* and computed shortest travel times are used to define the maximum acceptable travel time for each passenger. Using those maximum acceptable travel times for passengers as constraints, optimal routings are developed for each station using the SA algorithm.

$$\text{Degree of Circuity (DOC)} \geq \frac{(\text{Actual travel time})^i}{(\text{Shortest travel time})^i} \quad (1)$$

$$\text{Maximum Degree of Circuity (Max DOC)} \geq \max \left[\frac{(\text{Actual travel time})^i}{(\text{Shortest travel time})^i} \right] \quad (2)$$

i = Individual passenger

The buses at each station will serve the passengers who use that particular station. We developed the optimal routings and found the optimal number of buses for that operation. So, for each station and for each train, we calculate the number of buses for the operation and compare them with the available buses. If we have more buses than needed, then the surplus will be calculated, and the bus operation will be feasible. If the needed buses are more than the available buses, then the deficiency will be calculated, and it is necessary to examine whether there are surpluses at the adjacent stations for the relocation. If the total number of available buses at all the stations is more than the needed buses at all stations, then we may be able to make the entire service feasible with multiple relocations. Figure 1 shows the process for relocating the buses. In this algorithm, the relocated buses first serve alighting passengers at the station to deliver them to their destinations and then pick up passengers to take to the goal station to minimize the costs. In the algorithm of Figure 1, “s” is the index for stations and S is the index for the last station.

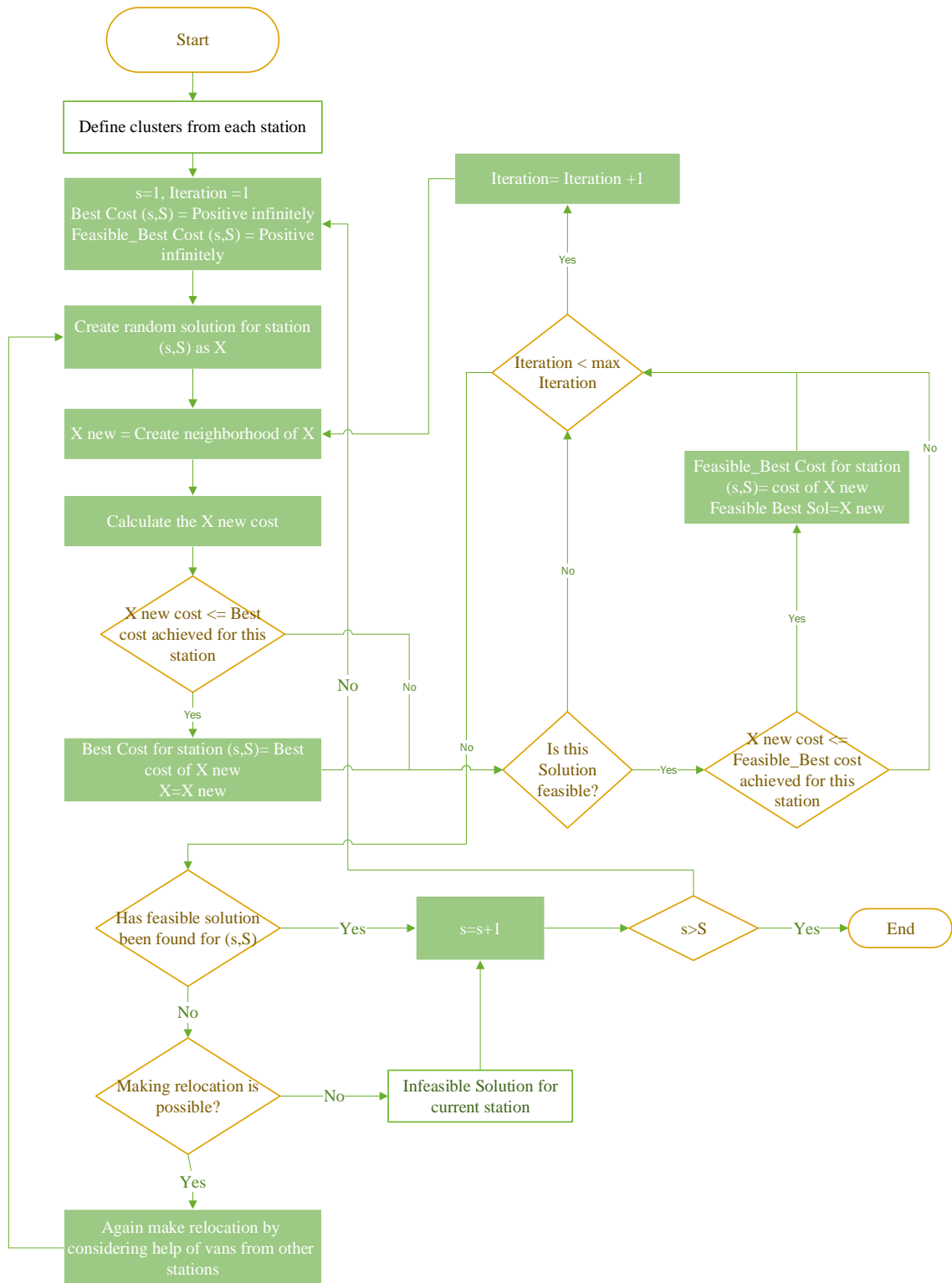


Figure 1. The conceptual flowchart for the proposed algorithm to solve the problem

To solve the problem in the model, in this study we applied the SA algorithm, one of the most efficient metaheuristics approaches for solving the vehicle routing problem with time windows (Wang C., Mu, Zhao, & Sutherland, 2015; Montoya-Torres, et al., 2015; Bräysy & Gendreau, 2005), because the local search strategy of the SA is more robust in finding high-quality solutions in VRP problems (Rochat & Taillard, 1995). The SA algorithm uses a probabilistic acceptance strategy which includes many iteration stages to find the global optimum by a temperature-changing schedule (Adewole, Otubamowo, Egunjobi, & Ng, 2012). The strong point of SA as opposed to other similar metaheuristics is that SA can easily escape from local minima and jump in the solution space to find a global optimum solution because the SA even accepts worse answers with a specific probability (Wang C. , Mu, Zhao, & Sutherland, 2015). The metaheuristics were coded in MATLAB 2016a. The code shown in Figure 2 describes the steps in the SA algorithm as applied to solve the proposed model. Following is the mathematical formulation. The objective function of the model is minimization of the total costs including passengers' travel costs and operating costs.

Variables:

$$v_k = \begin{cases} 1 & \text{vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{passenger } i \text{ is served with vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{ijk} = \begin{cases} 1 & \text{passenger } j \text{ is served after passenger } i \text{ with vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$r_i = \begin{cases} 1 & \text{passenger } i \text{ starts trip from train station (boarding)} \\ 0 & \text{passenger } i \text{ ends trip at train station (alighting)} \end{cases}$$

$$D_{ik} = \text{distance travelled up to passenger } i \text{ by vehicle } k$$

$TotalD_k =$ total distance travelled by vehicle k

$AT_i =$ Vehicle arrival time to passenger i node

$WT_i =$ In vehicle travel time of passenger i

$UC_i =$ Used capacity of vehicle after serving passenger i

$IC_i =$ number of passengers get on at train station on vehicle k

Objective function:

$$z = \min \sum_{i=1}^I C_T * WT_i + \sum_{k=1}^K C_O * TotalD_k \quad (3)$$

Constraints:

$$\sum_{k=1}^K y_{ik} = 1 \quad i = 1.2. \dots .I \quad (4)$$

$$\sum_{i=1}^I y_{ik} \leq M * v_k \quad k = 1.2. \dots .K \quad (5)$$

$$\sum_{k=1}^K v_k \leq \text{Available vehicles number} \quad (6)$$

$$2 * \alpha_{ijk} \leq (y_{ik} + y_{jk}) \quad i, j = 1.2. \dots .I; i \neq j, k = 1.2 \dots .K(7)$$

$$\sum_{k=1}^K \sum_{j=1}^I \alpha_{ijk} + \sum_{k=1}^K \alpha_{i0k} \geq 1 \quad i = 1.2. \dots I \quad (8)$$

$$\sum_{k=1}^K \sum_{i=1}^I \alpha_{ijk} + \sum_{k=1}^K \alpha_{0jk} \geq 1 \quad j = 1.2. \dots I \quad (9)$$

$$D_{jk} \geq D_{ik} - M(1 - \alpha_{ijk}) + d_{ij} \quad i, j = 1.2. \dots I; i \neq j. k = 1.2 \dots K \quad (10)$$

$$D_{ik} \geq d_{i0} y_{ik} \quad i = 1.2. \dots I \quad k = 1.2 \dots K \quad (11)$$

$$AT_i = \sum_{k=1}^K \frac{D_{ik}}{\text{speed}} \quad i = 1.2. \dots I \quad (12)$$

$$WT_i = r_i AT_i + (1 - r_i) \left(\frac{\text{TotalD}_k}{\text{speed}} - AT_i \right) \quad i = 1.2. \dots I \quad (13)$$

$$AT_i \leq \text{DOC} * \frac{d_{i0}}{\text{speed}} \quad i = 1.2. \dots I \quad (14)$$

$$AT_i + \frac{d_{i0}}{\text{speed}} \leq \text{cycle time} \quad i = 1.2. \dots I \quad (15)$$

$$UC_i = \sum_{k=1}^K IC_k y_{ik} + \sum_{k=1}^K \sum_{j=1}^I r_i \alpha_{jik} - \sum_{k=1}^K \sum_{j=1}^I (1 - r_i) \alpha_{jik} \quad (16)$$

$$IC_k = \sum_{j=1}^I y_{ik} r_i \quad k = 1.2. \dots K \quad (17)$$

$$UC_i \leq C \quad i = 1.2. \dots I \quad (18)$$

$$\text{Total}D_k \geq \sum_{k=1}^K (D_{ik} + d_{i0}y_{ik}) \quad i = 1.2. \dots I \quad (19)$$

$$\sum_{j=1}^I \alpha_{0jk} = v_k \quad k = 1.2 \dots K \quad (20)$$

$$\sum_{i=1}^I \alpha_{i0k} = v_k \quad k = 1.2 \dots K \quad (21)$$

$$r_i = (0.1), v_k = (0.1), \alpha_{ijk} = (0.1), y_{ik} = (0.1)$$

$$D_{ik} \geq 0. \text{Total}D_k \geq 0. AT_i \geq 0. WT_i \geq 0. UC_i \geq 0. IC_i \geq 0$$

Where:

I: number of passengers

K: number of available vehicles

d_{ij} : direct distance between passengers i and j

d_{i0} : direct distance between passenger i and station

C_T : time value of passenger per hour

C_O : unit operating cost of vehicle per kilometer

Speed: vehicles speed

DOC: Degree of Circuity

cycle time=20 minutes

C=capacity of vehicles

M: a big enough number used for modelling the expression

Formula (3) is the objective function of the problem. Constraint (4) specifies that each passenger is served by exactly one vehicle. Constraint (5) ensures that if a passenger is assigned to a vehicle, it is considered as a used vehicle. Eq. (6) makes sure that the total number of used vehicles does not exceed the total number of available vehicles. Eq. (7) defines that each path belongs to a single vehicle. Eq. (8) and (9) make sure that each passenger is assigned to a path. Eq. (10), (11), and (12) define the arrival time of vehicles to passengers. Eq. (13) calculates waiting time for passengers and Eq. (14) is additional time ratio constraint. Constraint (15) is the cycle time constraint. Eq. (16), (17), and (18) are capacity constraints. Total travelled distance is defined by Eq. (19). Eq. (20) and (21) ensure that every route starts and ends at the station.

3.2. Hypothetical Network

A hypothetical rail transit line that has four stations was developed to test the developed algorithm and demonstrate its efficiency. Also, we tested four trains to test and demonstrate the relocation of buses as shown in Figure 3.

In this example, the headway of the train is assumed to be 20 minutes and the travel time between two stations is assumed to be two minutes. The capacity for each bus is assumed to be eight passengers. We also waived passengers' boarding and alighting time at the nodes and the stations. Table 1 shows the number of boarding and alighting (B/L) passengers for each station and each train. For example, in Station 1 and for Train 1, 16 passengers need to be picked up and get on, and 24 passengers get off and need to be at Station 1 for Train 1. We also assumed that the average speed for feeder buses is 30 km/h and for trains is 60 km/h, and the distance between stations is 2 km. The travel time monetary value for each passenger has been placed at \$20 per hour, and \$0.3 per kilometers for vehicles is used as the operating cost. The origins and the destinations of the boarding and alighting passengers are randomly generated around the rail line for four trains. Figure 4 shows the boarding (blue points) and alighting passengers (red points) around the four stations for the four trains (yellow points).

Step 0: Initialization:
Set $s=1$, Best Cost=*positive infinite*, $T=T_0$, $\alpha=0.99$, previous station
help=0, next station help=0, vehicle (s ; $s: 1$ to S) =4, $\min \text{vehicle}(s; s: 1 \text{ to } S) = 0$

Step 1: Clustering:
Define passenger's cluster

Step 2: Create random solution
Considering the length of trip (number of passengers (s) +vehicles(s)-1)
set x as a random solution

Step 3: Find optimal solution:
IF $It1 < It1_{max}$, **THEN**
go to **step 4**, otherwise go to **step 6**
END IF

Step 4: IF $It2 < It2_{max}$, **THEN**
go to *step 4.1*, otherwise go to **step 5**
END IF

Step 4.1: Creating neighborhood:
set x_{new} = a neighborhood of x

Step 4.2: IF best cost for $x <$ best cost for x_{new} , **THEN**
set $x=x_{new}$ and go to *step 4.5*, otherwise go to *step 4.3*
END IF

Step 4.3: $p = \exp(-(\text{cost } x_{new} - \text{cost } x)/T * \text{Cost } x)$

Step 4.4: Accept $x = x_{new}$ by p -probability and reject- and $x = x_{new}$ by $(1-p)$ and go to *step 4.5*

Step 4.5: Cost calculation for x_{new}

Step 4.6: IF best cost for $x_{new} >$ best cost, **THEN**
set $bestsol = x_{new}$
END IF

Step 4.7: IF x_{new} is feasible (considering time ratio), and best cost for $x_{new} >$ feasible_best cost, **THEN**
set $feasible_bestsol = x_{new}$
END IF

Step 4.8: Reducing the temperature:
set $T = \alpha * T_0$ ($0 < \alpha < 1$)

Step 4.9: set $It2 = It2 + 1$ and go to **step 4**

Step 5: Set $It1 = It1 + 1$ and go to **step 3**

Step 6: IF $feasible_bestsol$ is empty, **THEN**
 $\min \text{vehicle}(s) = \text{vehicle}(s) + 1$ and go to **step 7**, otherwise go to **step 14**
END IF

Step 7: Calculate the following proportion for stations $s-1$ and $s+1$: number of passengers (s)/vehicle(s)

Step 8: IF $s-1$ exists and $\text{vehicle}(s-1) > \min \text{vehicle}(s-1)$, **THEN**
go to **step 9**, otherwise go to **step 11**
END IF

Step 9: IF proportion for station s is \leq the proportion for station $s+1$ or $\text{vehicle}(s+1) \leq \min \text{vehicle}(s+1)$
go to **step 10**, otherwise go to **step 11**
END IF

Step 10: Set previous station help (s)= previous station help (s)+1 and $\text{vehicle}(s-1) = \text{vehicle}(s-1) - 1$, $s = s - 1$, and go to **step 2**

Step 11: IF $s+1$ exists and $\text{vehicle}(s+1) > \min \text{vehicle}(s+1)$ **THEN**
go to **step 12**, otherwise go to **step 13**
END IF

Step 12: Set next station help (s)= next station help(s)+1 and $\text{vehicle}(s+1) = \text{vehicle}(s+1) - 1$ and go to **step 2**

Step 13: Show "The problem is not feasible; more vehicles is needed"

Step 14: IF $s < S$, **THEN**
set $s = s + 1$ and go to **step 2**, otherwise go to **step 15**
END IF

Step 15: Show results

Step 16: END

Figure 2. The developed SA algorithm to solve the model

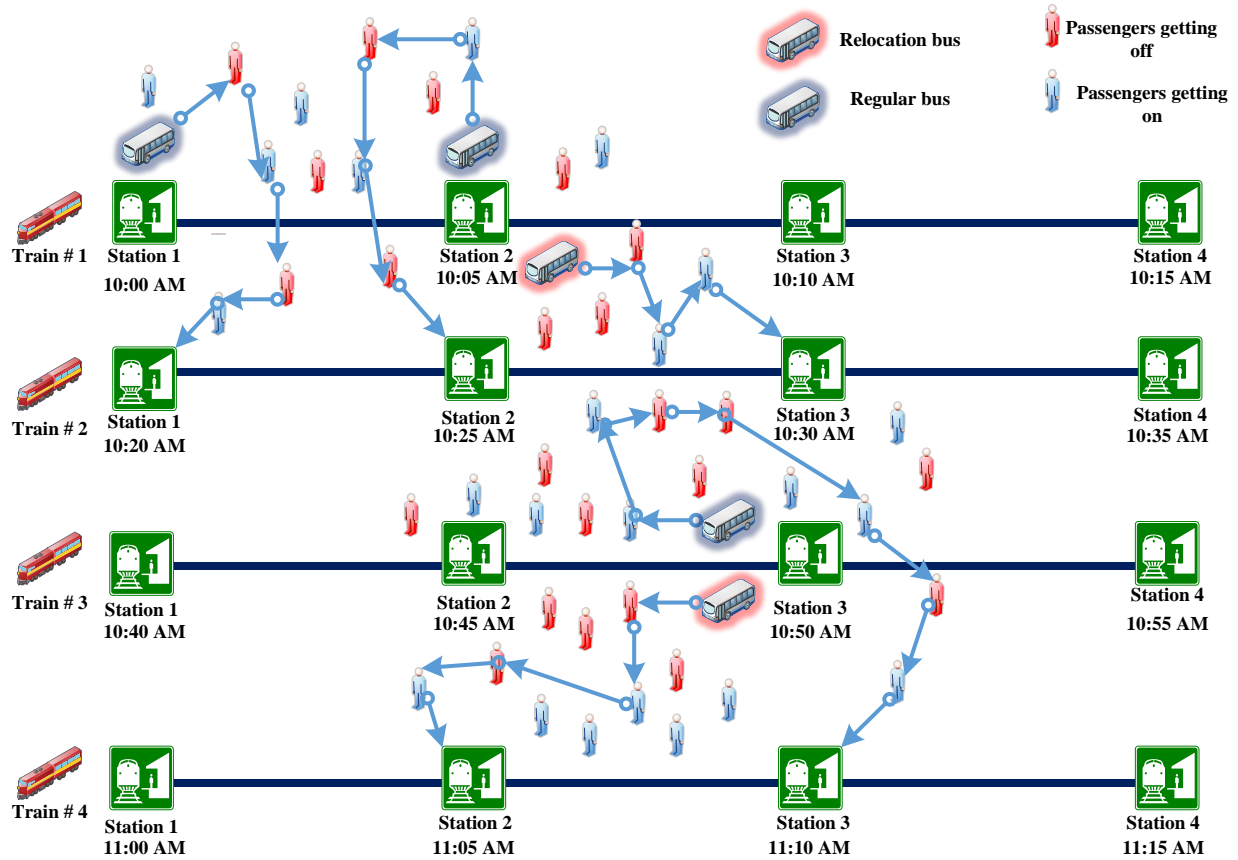


Figure 3. Conceptual operation of feeder transit (regular and relocation buses)

Table 1. Passenger information for each station and each train

Train	Station 1		Station 2		Station 3		Station 4		Average total direct travel distance (Km)
	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	
Train 1	16 + 24	2.08	21 + 19	2.33	19 + 21	2.23	25 + 15	2.08	2.18
Train 2	8 + 12	2.37	24 + 36	2.49	26 + 24	2.35	16 + 14	2.20	2.38
Train 3	16 + 29	2.37	12 + 13	2.15	26 + 29	2.37	15 + 20	2.26	2.31
Train 4	19 + 21	2.34	23 + 17	2.24	21 + 19	2.36	20 + 20	2.39	2.33
Average passenger total direct travel distance (Km)									2.30

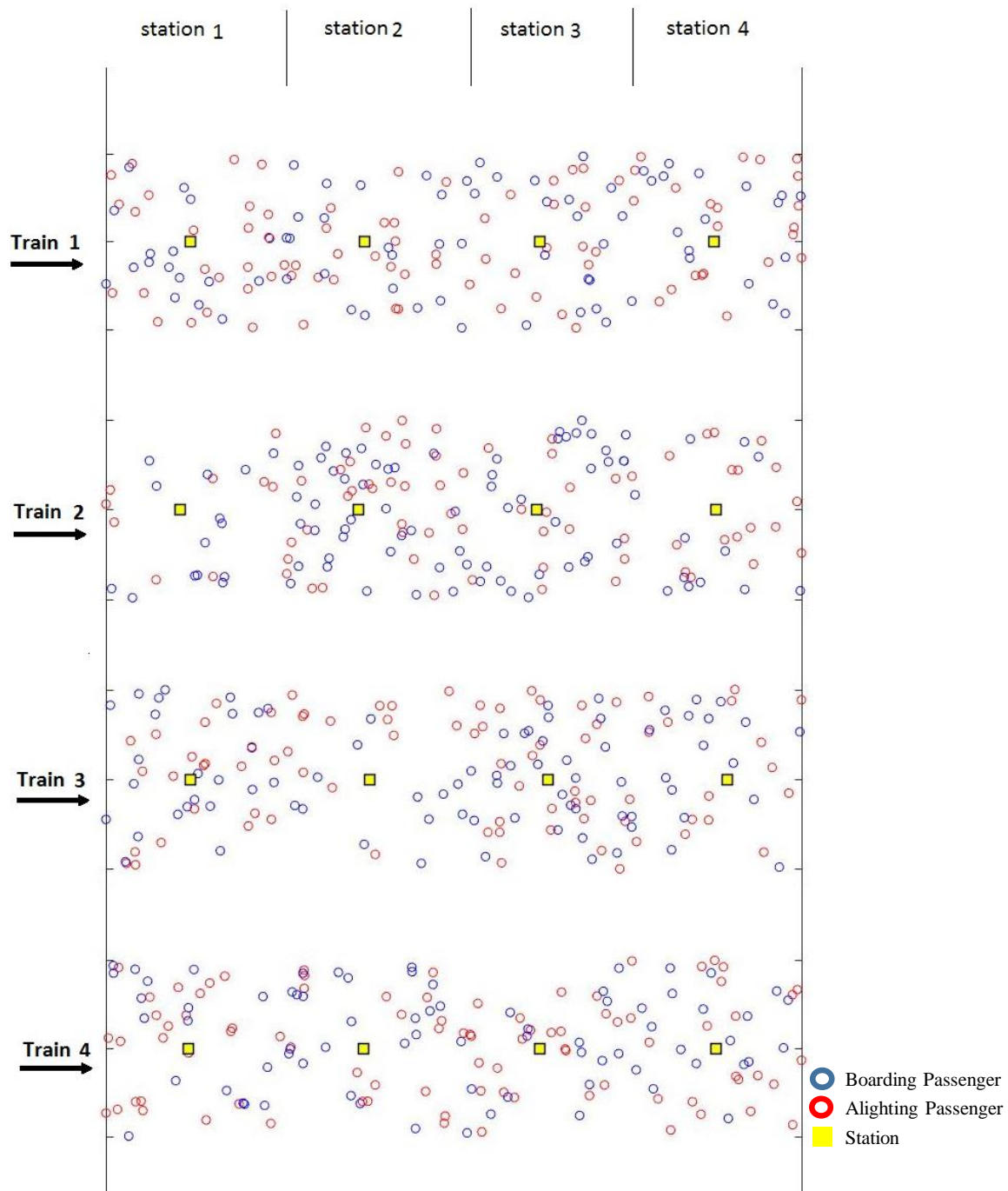


Figure 4. Geographical distributions of the passengers

4. ANALYSES AND RESULTS

The proposed SA started with the initializing of inputs and clustering of the passengers. The inputs are: passengers demand coordination 20 minutes before train arrival, vehicle speed, trains' schedules, stations' coordination, and velocity of trains. In the next step, the algorithm finds the optimal solution. It is important in this algorithm that the cost calculation process includes three parameters: without help, with help from the previous station, and help from next station. Finally, the outputs would be passenger's travel times, vehicles traveled distance, assigned buses in each station in each time window, relocated buses, and routes.

Figure 5 shows the results of the feeder bus routings including relocation of the buses. For Train 3, two buses were relocated, shown in dash lines. One bus was relocated from Station 2 to Station 1, and one bus was relocated from Station 2 to Station 3.

As mentioned previously, one aspect that can be distinguished in this research is considering individual passengers' Max *DOC* (Maximum Degree of Circuity). Unlike package delivery and pickup, passengers are likely to consider their travel times in the feeder bus to choose this service. So in this research, we calculated each individual passenger's shortest direct travel time from the origin to the station (or to the destination from the station), and computed the maximum acceptable travel time in the feeder bus as a constraint for the algorithm. Those acceptable additional times are calculated and used as a ratio (travel time in the feeder bus/direct travel time to the origin or to the station). Four different Max *DOCs* (2.5, 3, 3.5 and 4) were applied to the algorithm and the results are shown in Table 2 to Table 5.

Table 6 summarizes the results of Table 2 and shows the comparison of models with four Max *DOCs*. As expected, with higher Max *DOCs*, total costs of the service decreased due to the more relaxed constraint (from \$541.13 to \$539.00 to \$537.87 to \$536.28). With the higher ratio, the minimum number of buses used and vehicle traveled distance also decreased (from 259.55 km to 257.80 km to 255.60 km to 247.24 km) while passengers' total travel time more likely increased, although it did not show a clear relationship with the Max *DOC*.

The expectation of the shorter passenger travel time with the lower Max *DOC* due to a more direct route was not always fulfilled because of the less efficient routings to satisfy the lower maximum additional individual passengers' travel time ratio constraint. Contrary to the authors' initial intuition, which expected the longer passenger travel time and shorter operating distance due to the more circuitous routings with the higher Max *DOC*, no clear trade-off relationship was found between passenger travel time and vehicle operating costs along the various Max *DOC*s.

Although total costs were reduced with the higher Max *DOC* due to the more relaxed constraint for the algorithm, the savings in total costs were not significant (0.37%, 0.58% and 0.87%, respectively). So, transit agencies will decide whether to choose lower maximum additional individual passenger travel time ratio (2.5) for less maximum Degree of Circuity for individual passengers (2.49) and higher total costs (\$541.13) and longer vehicle operating distance (259.55 km) or higher maximum Degree of Circuity ratio (4.0) for higher maximum Degree of Circuity for individual passengers (3.76) but less total costs (\$536.28) and shorter vehicle operating distance (247.24 km).

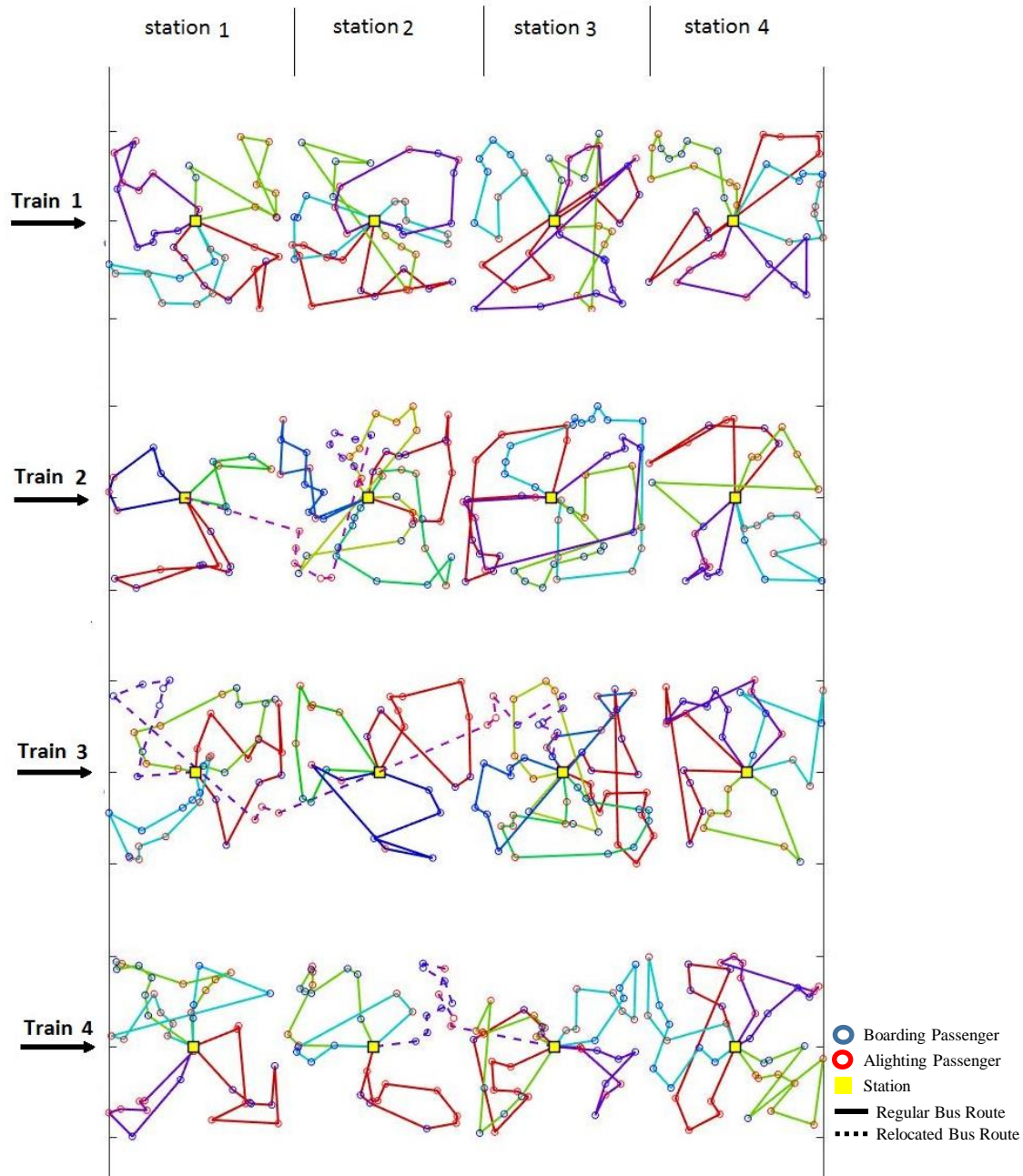


Figure 5. Results of the feeder bus routings (Max DOC of 2.5)

Table 2. Results of the model with Max *DOC* of 2.5

Train	Station	Boarding passengers	Alighting passengers	Total passengers	Available buses	Used bus trips	Relocation buses (+,-)	Vehicle traveled distance (Km)	Total passenger travel time (hour)	Average distance traveled to each station (Km)	Total passenger average distance traveled (Km)	Total cost in each stage (\$)	Total cost (\$)
#1	#1	17	23	40	4	4	0	15.19	1.47	1.65	1.64	33.93	136.29
	#2	19	21	40	4	4	0	16.91	1.41	1.58		33.19	
	#3	21	19	40	4	4	0	17.86	1.54	1.73		36.15	
	#4	18	22	40	4	4	0	15.96	1.41	1.59		33.03	
#2	#1	13	7	20	4	3	-1	9.47	0.69	1.55	1.61	16.58	132.72
	#2	31	29	60	4	5	1	21.85	1.95	1.46		45.61	
	#3	32	18	50	4	4	0	16.20	2.07	1.86		46.21	
	#4	12	18	30	4	4	0	12.52	1.03	1.54		24.33	
#3	#1	24	21	45	3	4	1	17.90	1.75	1.75	1.68	40.39	140.62
	#2	11	14	25	5	3	-2	13.73	0.99	1.78		23.84	
	#3	31	29	60	4	5	1	24.79	2.23	1.67		51.96	
	#4	16	14	30	4	4	0	13.64	1.02	1.53		24.43	
#4	#1	17	23	40	4	4	0	15.39	1.38	1.56	1.58	32.30	131.50
	#2	22	18	40	3	4	1	15.45	1.48	1.66		34.21	
	#3	17	23	40	5	4	-1	14.02	1.26	1.42		29.44	
	#4	18	22	40	4	4	0	18.66	1.50	1.68		35.54	
Total		319	321	640	64	64	0	259.55	23.16		1.63	541.13	541.13

Table 3. Results of the model with Max *DOC* of 3

Train	Station	Boarding passengers	Alighting passengers	Total passengers	Available buses	Used bus trips	Relocation buses (+,-)	Vehicle traveled distance (Km)	Total passenger travel time (hour)	Average distance traveled to each station (Km)	Total passenger average distance traveled (Km)	Total cost in each stage (\$)	Total cost (\$)
#1	#1	17	23	40	4	4	0	18.94	1.40	1.58	1.62	33.73	135.36
	#2	19	21	40	4	4	0	16.46	1.39	1.57		32.78	
	#3	21	19	40	4	4	0	16.65	1.54	1.73		35.82	
	#4	18	22	40	4	4	0	15.96	1.41	1.59		33.03	
#2	#1	13	7	20	4	4	0	9.95	0.58	1.31	1.67	14.64	138.23
	#2	31	29	60	4	4	0	23.56	2.31	1.73		53.26	
	#3	32	18	50	4	4	0	19.90	2.00	1.80		46.00	
	#4	12	18	30	4	4	0	12.52	1.03	1.54		24.33	
#3	#1	24	21	45	4	4	0	15.93	1.65	1.65	1.64	37.68	136.34
	#2	11	14	25	4	4	0	12.68	0.82	1.47		20.12	
	#3	31	29	60	4	4	0	22.32	2.37	1.78		54.12	
	#4	16	14	30	4	4	0	13.64	1.02	1.53		24.43	
#4	#1	17	23	40	4	4	0	15.39	1.38	1.56	1.56	32.30	129.07
	#2	22	18	40	4	4	0	12.85	1.47	1.65		33.17	
	#3	17	23	40	4	4	0	14.02	1.26	1.42		29.43	
	#4	18	22	40	4	4	0	17.03	1.45	1.63		34.17	
Total		319	321	640	64	64	0	257.80	23.08		1.62	539.00	539.00

Table 4. Results of the model with Max *DOC* of 3.5

Train	Station	Boarding passengers	Alighting passengers	Total passengers	Available buses	Used bus trips	Relocation buses (+,-)	Vehicle traveled distance (Km)	Total passenger travel time (hour)	Average distance traveled to each station (Km)	Total passenger average distance traveled (Km)	Total cost in each stage (\$)	Total cost (\$)
#1	#1	17	23	40	4	4	0	18.94	1.40	1.58	1.62	33.73	135.36
	#2	19	21	40	4	4	0	16.46	1.39	1.57		32.78	
	#3	21	19	40	4	4	0	16.65	1.54	1.73		35.82	
	#4	18	22	40	4	4	0	15.96	1.41	1.59		33.03	
#2	#1	13	7	20	4	4	0	9.95	0.58	1.31	1.66	14.64	137.27
	#2	31	29	60	4	4	0	22.22	2.28	1.71		52.30	
	#3	32	18	50	4	4	0	19.90	2.00	1.80		46.00	
	#4	12	18	30	4	4	0	12.52	1.03	1.54		24.33	
#3	#1	24	21	45	4	4	0	15.93	1.65	1.65	1.64	37.68	136.34
	#2	11	14	25	4	4	0	12.68	0.82	1.47		20.12	
	#3	31	29	60	4	4	0	22.32	2.37	1.78		54.12	
	#4	16	14	30	4	4	0	13.64	1.02	1.53		24.43	
#4	#1	17	23	40	4	4	0	15.39	1.38	1.56	1.57	32.30	128.90
	#2	22	18	40	4	4	0	12.85	1.47	1.65		33.17	
	#3	17	23	40	4	4	0	14.02	1.26	1.42		29.43	
	#4	18	22	40	4	4	0	16.17	1.46	1.64		33.99	
Total		319	321	640	64	64	0	255.60	23.06		1.62	537.87	537.87

Table 5. Results of the model with Max *DOC* of 4

Train	Station	Boarding passengers	Alighting passengers	Total passengers	Available buses	Used bus trips	Relocation buses (+,-)	Vehicle traveled distance (Km)	Total passenger travel time (hour)	Average distance traveled to each station (Km)	Total passenger average distance traveled (Km)	Total cost in each stage (\$)	Total cost (\$)
#1	#1	17	23	40	4	4	0	18.94	1.40	1.58	1.62	33.73	135.36
	#2	19	21	40	4	4	0	16.46	1.39	1.57		32.78	
	#3	21	19	40	4	4	0	16.65	1.54	1.73		35.82	
	#4	18	22	40	4	4	0	15.96	1.41	1.59		33.03	
#2	#1	13	7	20	4	4	0	9.95	0.58	1.31	1.67	14.64	137.09
	#2	31	29	60	4	4	0	22.22	2.28	1.71		52.30	
	#3	32	18	50	4	4	0	16.16	2.05	1.84		45.82	
	#4	12	18	30	4	4	0	12.52	1.03	1.54		24.33	
#3	#1	24	21	45	4	4	0	15.93	1.65	1.65	1.64	37.68	134.94
	#2	11	14	25	4	4	0	12.68	0.82	1.47		20.12	
	#3	31	29	60	4	4	0	17.70	2.37	1.78		52.71	
	#4	16	14	30	4	4	0	13.64	1.02	1.53		24.43	
#4	#1	17	23	40	4	4	0	15.39	1.38	1.56	1.57	32.30	128.90
	#2	22	18	40	4	4	0	12.85	1.47	1.65		33.17	
	#3	17	23	40	4	4	0	14.02	1.26	1.42		29.43	
	#4	18	22	40	4	4	0	16.17	1.46	1.64		33.99	
Total		319	321	640	64	64	0	247.24	23.11		1.62	536.28	536.28

Table 6. Summary of the routings for the various circuities

Models	Passenger related factors						Agency related factors				Total factors	
	Total passenger average direct travel distance (Km)	Total passenger average distance traveled (Km)	Circuitry of passenger travels due to feeder bus routings	Maximum applied circuitry for passengers	Average passengers total travel time (h)	Total passenger travel cost (\$)	Total bus trips	Average vehicles traveled distance (Km)	Total vehicles traveled distance (Km)	Total vehicle operating cost (\$)	Total Cost (\$)	Percentage change of the total cost (%)
With ratio of 2.5	0.77	1.63	2.12	2.49	1.45	463.27	64	4.05	259.55	77.87	541.13	-
With ratio of 3	0.77	1.62	2.11	2.99	1.44	461.67	64	4.03	257.8	77.34	539	-0.37
With ratio of 3.5	0.77	1.62	2.11	3.21	1.44	461.19	64	3.99	255.6	76.68	537.87	-0.58
With ratio of 4	0.77	1.62	2.11	3.76	1.44	462.11	64	3.86	247.24	74.17	536.28	-0.87

5. IMPACT OF AUTOMATION IN FEEDER TRANSIT NETWORK

Demand responsive feeder transit which serves first-mile and last-mile are usually small-sized bus services. Although they are not new, they have been widely discussed in recent years due to the technological innovation and recent successful business models by transportation network companies (TNCs), such as Uber and Lyft. People have become familiar with smartphone apps to request service, and it has become much easier to dispatch flexible routing small-sized buses with the better communication technology and improved optimization process.

One inherent problem in using the small-sized bus for the flexible service is the high labor costs out of the total costs because of the lower vehicle capacity. Although total costs would be lower than a regular size bus for the same frequencies and routings due to the small size of the vehicles, most likely more frequent service or more vehicles would be required, possibly raising total costs because of lower vehicle capacity. Recent development of the automated vehicle technology can be a solution for the smaller vehicle flexible transit service's higher portion of labor costs to the total costs, and it can mostly eliminate or minimize the drivers' labor costs.

Furthermore, lower operating costs may improve transit routing efficiency. In most studies, there are two major components in the objective function to optimize the transit network design, passenger travel time and operating costs. If operating costs are lowered, obviously total costs consisting of operating costs and passenger travel time costs will be reduced. Will lower operating costs change passenger travel times? Will they improve overall transit network efficiency?

In this study, the economic impact by the automation of the transit operation will be examined. Then, using the authors' previously developed algorithm for the optimal demand responsive flexible feeder bus network, optimal flexible transit networks will be generated for various operating costs, and their network characteristics will be examined and compared.

6. ESTIMATION OF TRANSIT OPERATING COSTS

Cost is an important factor in transit decision-making since many decisions are cost-related or cost-oriented. Reliable and accurate cost models and estimates lead to better operational and strategic decisions. Operational costs of a transit system are key factors in critical decisions such as implementation, development, and extension of a system. Some cost models/estimates are simple while others are sophisticated, complex and consist of several different variables, but usually the outputs are expressed as dollars (or other currencies) per some units (e.g., per distance units, per time units, and per operator). However, regardless of the complexity and sophistication of the models, they should be based on valid and complete sources, cost components, and they should be validated (On Target Performance Group, 2011). There are usually some fixed costs that are unaffected by the amount of service and some variable costs that increase by vehicle mileage (Victoria Transport Policy Institute, 2016).

6.1 Conventional Bus Operating Cost

In this section, a conventional bus is defined as a bus driven by a human driver on a bus route.

The main operating cost functions that have been considered by the National Transit Database (NTD) of the Federal Transit Administration (FTA) are as follows (Federal Transit Administration (FTA), 2018):

- Vehicle operations
- Vehicle maintenance
- Facility maintenance
- General administration

However, the main operating cost types that have been considered by the National Transit Database (NTD) of the Federal Transit Administration (FTA) are as follows (Federal Transit Administration (FTA), 2018):

- Operators' salaries and wages
- Other salaries and wages

- Fringe benefits
- Service costs
- Fuel and lubricants
- Tires and tubes
- Other materials and supplies
- Utilities
- Casualty and liability costs
- Taxes
- Purchased transportation expenses
- Miscellaneous expenses

Similar cost structures have been used by different agencies. In a 2011 study in Denver, Colorado, the main cost components were identified as administrative costs and costs based on scheduled service miles; however, the following items were allocated separately to account for differences in service class or the type of vehicles used on the routes (On Target Performance Group, 2011):

- Operators (wages and benefits)
- Consumables (fuel, tires, oil, etc.)
- Parts (parts needed in vehicle maintenance)
- Running repair (running repair mechanic hours, labor costs, etc.)

Table 7 summarizes the bus operating costs per kilometer in the US, UK, and Switzerland. The UK costs seemed to be based on different assumptions and variables. Due to the variations, some researchers have tried to categorize the operating costs. Table 8 demonstrates operating costs per revenue kilometer by agency size.

Table 7. Bus operating cost per kilometer

Location	Year	Total Operating Cost (\$)	Total Units (km)	Unit Cost (\$/km)	Unit Cost in 2018 (\$/km)	Source
US: Bus	2007	18,267,000,000	2,999,809,760	6.09	7.31	(Victoria Transport Policy Institute, 2016)
Full US Reporters	2016	20,516,800,000	2,869,614,154	7.15	7.46	(Federal Transit Administration (FTA), 2018)
Top 50 US Reporters	2016	14,818,200,000	1,700,428,644	8.71	9.09	(Federal Transit Administration (FTA), 2018)
MTA New York City Transit	2016	2,779,372,331	139,792,686	19.88	20.75	(Federal Transit Administration (FTA), 2018)
Massachusetts Bay Transp. Authority	2016	412,610,862	37,167,234	11.10	11.59	(Federal Transit Administration (FTA), 2018)
Southeastern Pennsylvania Transportation Authority	2016	628,216,161	64,041,234	9.81	10.24	(Federal Transit Administration (FTA), 2018)
Chicago Transit Authority	2016	801,281,245	84,176,213	9.52	9.94	(Federal Transit Administration (FTA), 2018)
Washington Met Area Transit Auth.	2016	590,647,746	63,349,542	9.32	9.73	(Federal Transit Administration (FTA), 2018)

Location	Year	Total Operating Cost (\$)	Total Units (km)	Unit Cost (\$/km)	Unit Cost in 2018 (\$/km)	Source
King County Department of Transportation	2016	477,562,833	54,060,019	8.83	9.22	(Federal Transit Administration (FTA), 2018)
Maryland Transit Administration	2016	272,115,276	32,413,493	8.40	8.77	(Federal Transit Administration (FTA), 2018)
Tri-County Metropolitan Transportation District of Oregon	2016	251,249,183	33,311,352	7.54	7.87	(Federal Transit Administration (FTA), 2018)
New Jersey Transit Corporation	2016	956,997,264	128,337,237	7.46	7.79	(Federal Transit Administration (FTA), 2018)
New Orleans Regional Transit Authority	2016	62,560,998	8,573,260	7.30	7.62	(Federal Transit Administration (FTA), 2018)
Central Puget Sound Regional Transit Auth. DBA Sound Transit	2016	118,582,934	19,183,891	6.18	6.45	(Federal Transit Administration (FTA), 2018)

Location	Year	Total Operating Cost (\$)	Total Units (km)	Unit Cost (\$/km)	Unit Cost in 2018 (\$/km)	Source
City of Phoenix Public Transit Dept. dba Valley Metro	2016	147,701,121	26,318,168	5.61	5.85	(Federal Transit Administration (FTA), 2018)
Regional Transp District (Denver, CO)	2008	184,514,747	41,408,148	4.46	5.35	(On Target Performance Group, 2011)
Regional Transportation District (Denver, CO)	2009	184,358,322	40,234,308	4.58	5.34	(On Target Performance Group, 2011)
Jacksonville Transportation Authority	2016	71,581,487	14,022,097	5.10	5.32	(Federal Transit Administration (FTA), 2018)
Memphis Area Transit Authority	2016	41,583,335	8,499,314	4.89	5.10	(Federal Transit Administration (FTA), 2018)
Charlotte Area Transit System	2016	80,465,139	16,924,262	4.75	4.96	(Federal Transit Administration (FTA), 2018)
Central Oklahoma Transp. and Parking Auth.	2016	21,729,641	4,755,912	4.57	4.77	(Federal Transit Administration (FTA), 2018)

Location	Year	Total Operating Cost (\$)	Total Units (km)	Unit Cost (\$/km)	Unit Cost in 2018 (\$/km)	Source
Greater Roanoke Transit Company	2016	7,011,634	2,578,142	2.72	2.84	(Federal Transit Administration (FTA), 2018)
England outside London	2016	NA	NA	2.67	2.79	(Department for Transport (DfT), 2017)
Wales	2016	NA	NA	2.49	2.60	(Department for Transport (DfT), 2017)
Scotland	2016	NA	NA	2.41	2.52	(Department for Transport (DfT), 2017)
Great Britain outside London	2016	NA	NA	2.62	2.73	(Department for Transport (DfT), 2017)
Switzerland (Urban)	2011	NA	NA	7.21	8.05	Bundesamt für Verkehr (2011) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Switzerland (Regional)	2011	NA	NA	6.77	7.56	Bundesamt für Verkehr (2011) cited in (Bosch, Becker, Becker, & Axhausen, 2017)

Notes: Total Units (km) = "Annual Vehicle Revenue Kilometers." Unit Costs in 2018 calculated using "CPI Inflation Calculator" (Bureau of Labor Statistics (BLS), 2018). £ 1 = \$ 1.31 and CHF 1 = \$ 1.01 (July 25, 2018)

Table 8. Operating cost by agency size for conventional buses

Agency Size		Florida		Southeast US		US	
		2008 Costs	2018 Costs	2008 Costs	2018 Costs	2008 Costs	2018 Costs
All Agencies	<i>Per Revenue Kilometer</i>	\$ 4.49	\$ 5.37	\$ 4.31	\$ 5.16	\$ 6.08	\$ 7.29
	<i>Per Passenger Kilometer</i>	\$ 0.47	\$ 0.56	\$ 0.50	\$ 0.60	\$ 0.55	\$ 0.66
Small Agencies	<i>Per Revenue Kilometer</i>	\$ 2.88	\$ 3.45	\$ 2.65	\$ 3.18	\$ 3.30	\$ 3.95
	<i>Per Passenger Kilometer</i>	\$ 0.59	\$ 0.71	\$ 0.56	\$ 0.67	\$ 0.47	\$ 0.56
Medium-sized Agencies	<i>Per Revenue Kilometer</i>	\$ 3.52	\$ 4.23	\$ 4.09	\$ 4.90	\$ 4.32	\$ 5.18
	<i>Per Passenger Kilometer</i>	\$ 0.50	\$ 0.60	\$ 0.54	\$ 0.65	\$ 0.45	\$ 0.54
Large Agencies	<i>Per Revenue Kilometer</i>	\$ 4.84	\$ 5.80	\$ 4.69	\$ 5.62	\$ 6.95	\$ 8.33
	<i>Per Passenger Kilometer</i>	\$ 0.46	\$ 0.55	\$ 0.49	\$ 0.59	\$ 0.57	\$ 0.68

Source: (Reich & Davis, 2011)

Note: 2018 Costs calculated using “CPI Inflation Calculator” (Bureau of Labor Statistics (BLS), 2018)

There have been some modeling efforts to estimate bus operating costs such as a hybrid approach (combining a Bottom-Up and a Top-Down approach) for Italian local public bus transport sector (Avenali, Boitani, Catalano, D’Alfonso, & Matteucci, 2017) and a cost and fare estimation for the bus transit system of Santiago through a uniform price with and without subsidy and two-part tariffs with and without subsidy (Batarce & Galilea, 2018).

To conclude this section, due to several different factors – such as differences in wages, agency size, vehicle size, service features and design, demand, and other regional differences – bus operating cost varies from city to city and even from service to service, but the main costs are crew wages, capital costs,

maintenance, and administration costs. The National Transit Database (NTD) of the FTA has a comprehensive nationwide transit systems database with several different metrics that can be used to dig data for certain cities and/or aggregate level (e.g., by agency size, urban population, etc.). Based on the data, \$3.95 – \$8.33 is the range of operating cost per revenue kilometer for the US.

6.2 Expected Bus Operating Cost with Emerging Technology

There are various bus types with emerging technologies. Their definitions are as follows:

- CV Bus Transit: a human driver operates the bus on a bus route in a connected vehicle environment.
- AV Bus Transit: an autonomous bus operates on a bus route.
- CAV Bus Transit: an autonomous bus operates in a connected vehicle environment.

Table 9 summarizes the possible options of the future evolution of current transit with developing technologies. During the transitional period, some researchers consider semi-AV systems (Zhang, Jenelius, & Badia, 2018) and also different levels of market penetration. Shared transport services have also been proposed, discussed and analyzed in the recent literature.

The future of public transportation has been discussed in positive and negative ways in the recent literature. A recent study based on the Capital Metropolitan Transportation Authority bus fleet in Austin, Texas, predicts that electric buses will be life-cycle cost-competitive by 2022 and self-driving buses enabled by electric engines possibly could be adopted by 2023-2026 or 2024-2035, depending on different scenarios (Quarles & Kockelman, 2018).

Stocker & Shaheen (2017) discussed the existing business models of shared mobility providers.

The models were as follows:

- Business-to-Consumer: Vendors typically own/lease and maintain a fleet of vehicles and allow users to access these vehicles via membership and/or usage fees
- Peer-to-Peer: Companies supervise transactions among individual owners and renters by providing the necessary platform and resources needed for the exchange

- For-Hire: A customer or passenger hires a driver on an as-needed basis for transportation services.

Table 9. Possible options of the evolution of current transit in the future with developing technologies.

Transit System	CV	EV	Note
Conventional	x	x	Existing condition in many cities.
	x	✓	
	✓	x	
	✓	✓	
AV	x	x	
	x	✓	
	✓	x	
	✓	✓	

Notes: CV: Connected Vehicle; EV: Electric Vehicle; AV: Autonomous

Stocker & Shaheen (2017) stated that SAV (Shared Automated Vehicles) business models would depend on vehicle ownership and network operations and discussed the possibilities of each of the aforementioned business models.

Bosch et al. (2017) stated that public transportation (in its current form) would only remain economically competitive where demand can be bundled to larger units, which would be the case in dense urban areas, where public transportation can be cheaper than autonomous taxis (even if pooled) and private cars. In their study, they reviewed and listed past relevant studies and also estimated associated costs of emerging systems through a comprehensive approach. Table 10 summarizes the operating costs, mainly per passenger-distance.

Table 10. Estimated operating costs for some emerging transit technologies

System Type	Year	Cost	Currency	Cost Unit	Cost in 2018 US Dollars (\$/km)	Note	Source
Purpose-built shared AV system for a small to medium town	2013	0.09	\$	Per trip-km	0.10		Burns et al. (2013) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Shared AV system (cost of AVs only)	2015	0.31	\$	Per kilometer	0.33	Investment cost = \$70,000	Fagnant and Kockelman (2015) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Shared AVs	2015	0.27	\$	Per trip-km	0.29	Operating cost plus 30% profit margin	Johnson (2015) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Purpose-built shared AVs used as pooled taxis	2015	0.10	\$	Per trip-km	0.11		Johnson (2015) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Fully autonomous vehicles used with ride-sharing	2016	0.12 - 0.19	\$	Per passenger-km	0.13 - 0.20	Lower-bound and upper-bound costs	Stephens et al. (2016) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Ride-sharing scheme in an urban area in Germany	2016	0.15	€	Per passenger-km	0.18		Friedrich and Hartl (2016) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Fully autonomous vehicles used with ride-sharing	2016	0.19	\$	Per passenger-km	0.19		Johnson and Walker (2016) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Fully autonomous vehicles in a ride-sharing scheme	2016	0.09	€	Per passenger-km	0.11	Lower cost than rail services	Hazan et al. (2016) cited in (Bosch, Becker, Becker, & Axhausen, 2017)
Autonomous bus in urban area	2017	0.25	CHF	Per passenger-km	0.26		(Bosch, Becker, Becker, & Axhausen, 2017)
Autonomous bus in regional area	2017	0.42	CHF	Per passenger-km	0.43		(Bosch, Becker, Becker, & Axhausen, 2017)

Notes: 2018 Costs calculated using “CPI Inflation Calculator” (Bureau of Labor Statistics (BLS), 2018). € 1 = \$ 1.17 and CHF 1 = \$ 1.01 (July 25, 2018)

Bosch et al. (2017) stated that the overhead costs of shared services were neglected in all previous cases that they reviewed (all rows of Table 10 except the last two rows). This is a major limitation since the new service markets in the transport sector such as Uber and Lyft only offer overhead services rather than actual transportation services. In their comprehensive analysis, they accounted for potential impacts of electrification and automation on operating costs (Table 11); however, these impacts are predicted for solo (1 seat), midsize (4 seats), van (8 seats), and minibus (20 seats) vehicles. In addition, electrification and automation may decrease total operating cost per kilometer (of city and regional buses with 60 seats) by 5.5% and 55%, respectively (Bosch, Becker, Becker, & Axhausen, 2017). Since it's possible that the smaller vehicles might be used in public transport using emerging technologies, they provided detailed cost estimates for vans (8 seats) and minibuses (20 seats) along with regular buses that are presented in Table 12.

The Bosch et al. (2017) study results indicated that private vehicle ownership would remain attractive in comparison with other modes. Line-based public transportation will remain viable for high-demand relations (at dense urban areas); shared taxis and transit based on smaller vehicles would replace line-based public transportation on low-demand relations. One-seaters would be used for first- and last-mile connections if fleet heterogeneity would not be a problem, as was also studied and proposed by Chong et al. (2011). Cleaning cost would be an important cost component that might be as much as one-third of the total operating cost. It is usually an overlooked cost that may add \$0.50-1.00 per trip or 5-10¢ per vehicle-mile, plus travel time and costs for driving to cleaning stations (Litman, 2018).

Zhang et al. (2018) proposed an analytical cost model of bus operations considering emerging automation technology. The generalized cost (the sum of waiting, riding, operating and capital cost) was modeled for conventional (level 0), semi-autonomous (level 4) and fully autonomous (level 5) bus services on a generic corridor-and-branches network. The bus company cost consists of bus operating cost (80%) and overhead cost (20%). Crew cost (40% of total cost) is a major cost component. The main impact of automation will be in the direct reduction or elimination of on-vehicle crew costs. The CV environment will also contribute with platooning, which will reduce the operating costs of buses. The

main cost components considered in the study included fixed or time-based costs, such as travel time cost, vehicle hour cost and so on. The analytical and numerical results supported the fully autonomous bus services even with high additional capital costs; however, the success of semi-autonomous bus services was weak and very dependent on network, demand and other factors.

Table 11. Potential impacts of electrification and automation on operating costs

Item	Electric	Autonomous
Acquisition	-	+20%
Yearly insurance	-35%	-50%
Yearly tax	-100%	-
Yearly parking	-	-
Yearly toll	-	-
Maintenance	+28%	-
Cleaning	-	\$0.02-0.05 per kilometer
Tires	-	-10%
Fuel	-50%	-10%

Source: (Bosch, Becker, Becker, & Axhausen, 2017)

Table 12. Estimated operating costs considering different emerging technologies

Mode	Capacity	Technology	Urban		Regional	
			Per Vehicle-km	Per Passenger-km	Per Vehicle-km	Per Passenger-km
Bus	60	Conventional	\$ 7.37	\$ 0.55	\$ 6.92	\$ 0.92
		Electric	\$ 7.00	\$ 0.52	\$ 6.57	\$ 0.87
		Autonomous	\$ 3.32	\$ 0.25	\$ 3.12	\$ 0.41
		EV & AV	\$ 3.15	\$ 0.24	\$ 2.95	\$ 0.39
Minibus	20	Conventional	\$ 2.02	\$ 0.48	\$ 2.51	\$ 1.30
		Electric	\$ 1.95	\$ 0.47	\$ 2.44	\$ 1.27
		Autonomous	\$ 1.07	\$ 0.26	\$ 0.70	\$ 0.36
		EV & AV	\$ 1.01	\$ 0.25	\$ 0.63	\$ 0.32
Van	8	Conventional	\$ 3.66	\$ 1.75	\$ 2.42	\$ 1.25
		Electric	\$ 3.60	\$ 1.72	\$ 2.38	\$ 1.23
		Autonomous	\$ 0.80	\$ 0.38	\$ 0.60	\$ 0.31
		EV & AV	\$ 0.77	\$ 0.36	\$ 0.57	\$ 0.29

Source: (Bosch, Becker, Becker, & Axhausen, 2017)

Notes: Costs presented in 2018 US dollars. Shaded cells calculated based on 5.5% and 55% cost decrease of electrification and automation, respectively (Bosch, Becker, Becker, & Axhausen, 2017). Assumption: EV and AV influence independently and result in 57.25% decrease. 2018 Costs calculated using “CPI Inflation Calculator” (Bureau of Labor Statistics (BLS), 2018)

Presently, new transportation technologies are under development, such as AV and CV, while some other technologies are on the path to mass adoption, such as shared services and EV. Considering different rates of progress and adoption and the way these technologies may blend and interact, currently most of the predictions and scenarios are mixed with significant levels of speculation and contemplation. Some operating costs have been proposed and examined, but estimated costs may not be reliable due to uncertainty about the business model schema and also involved technologies and economies of scales phenomenon. However, to have rough estimates in 2018 costs, the values in Table 8 were adjusted by the 5.5% and 55% reductions proposed by Bosch et al. (2017) for electrification and automation of buses, respectively. For smaller transit vehicles such as vans and minibuses, the values in Table 12 may be used. It should be noted that estimates were based on data from Switzerland but converted into 2018 US dollars. Table 13 presents the estimates in 2018 costs by different agency sizes.

Table 13. Operating cost per revenue kilometer for buses by agency size for emerging technologies (2018 US dollars)

Agency Size	Conventional	Electric	Autonomous	EV & AV
All Agencies	\$ 7.29	\$ 6.92	\$ 3.28	\$ 3.12
Small Agencies	\$ 3.95	\$ 3.75	\$ 1.78	\$ 1.69
Medium-sized Agencies	\$ 5.18	\$ 4.92	\$ 2.33	\$ 2.22
Large Agencies	\$ 8.33	\$ 7.92	\$ 3.75	\$ 3.56

Notes: 2018 Costs calculated using “CPI Inflation Calculator” (Bureau of Labor Statistics (BLS), 2018).

Assumption: EV and AV influence independently and result in 57.25% decrease.

Based on the values in Table 8, Table 12, and Table 13, the following ranges of operating costs (2018 US dollars) per kilometer seem reasonable:

- Bus (conventional): \$3.95 – 8.33
- Bus (emerging technologies): \$1.69 – 3.56
- Minibus (conventional): \$2 – 2.5

- Minibus (emerging technologies): \$0.63 – 1.01
- Van (conventional): \$2.42 – 3.66
- Van (emerging technologies): \$0.57 – 0.77

7. ANALYSES AND RESULTS WITH VARIOUS TRANSIT OPERATING COSTS

A hypothetical rail transit line that has four stations was created to test the developed algorithm and demonstrate its efficiency. Also, we tested four trains to test and demonstrate the relocation of buses. In this example, the capacity for each bus is assumed to be eight passengers. We also waived passengers' boarding and alighting time at the nodes and the stations. Table 14 shows the number of boarding and alighting passengers for each station and for each train. For example, in Station 1, 20 passengers need to be picked up and get on and 20 passengers get off and need to be at Station 1, and so on. The bus speed is assumed to be 30km/h and the travel time value for passengers is assumed to be \$20/hour. The origins and destinations of the boarding and alighting passengers are randomly generated around the rail line for four trains. Figure 6 shows the boarding (blue points) and alighting passengers (red points) around four stations.

Table 14. Passenger information for each station

Station 1		Station 2		Station 3		Station 4		Average total direct travel distance (Km)
Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	Boarding/Alighting passengers (prs)	Average direct travel distance (Km)	
20+20	1.1714	19+16	1.1688	18+22	1.1962	18+17	1.2765	1.2019

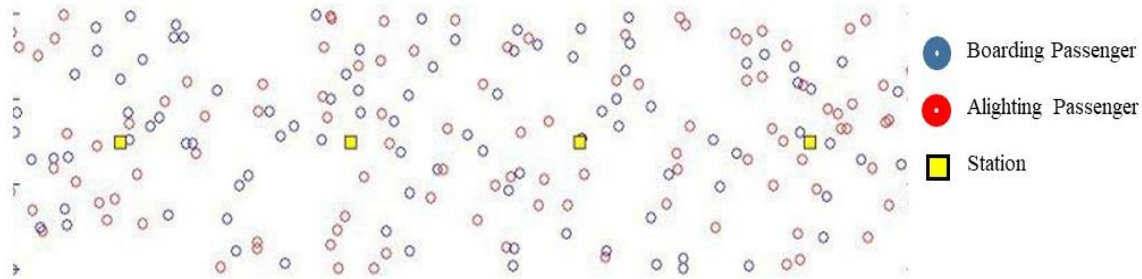


Figure 6. Geographical distributions of the passengers

The generated demand on the hypothetical network is applied to the optimization algorithm. The key assumptions in the computation are bus speed of 30km/hr to convert passenger travel distance to their travel time, and \$20/hr time value to convert their travel time to travel costs. As discussed in the literature review, estimating vehicle operating costs per distance requires wild estimations with many assumptions because the components for vehicle operating costs are in different dimensions, such as fixed costs, variable costs, time-related, distance-related, etc. Cost estimation for vehicle operation with emerging technology requires even wilder assumptions and guesses. So, the authors tested five operating cost scenarios, from \$1/km to \$5/km, which covers CAV van or minibus services to general van or minibus services, as a sensitivity analysis to find the impact of operating cost reduction on the optimal feeder transit network. Figure 7 shows the sample optimal network with \$3/km operating costs.

Table 15 shows the network characteristics of five unit operating costs. Obviously, when unit operating costs decline (\$5/km to \$1/km), total operating costs (\$395.85 to \$90.60) and total costs (\$640.66 to \$302.91) decline. One thing that should be noted is when unit operating costs decline (\$5/km to \$1/km), average passenger travel distance (3.44 km to 2.99 km) and total passenger travel costs (\$244.82 to \$212.32) decline while total operating costs per unit operating costs increase (79.17 to 90.60). Also, the total costs decrease with the lower unit operating costs even if the networks remain the same (total cost with \$5/km network), due to the lower total operating costs. However, their total costs are always higher than the total costs of the optimized network, because the feeder network with \$5/km unit

cost was not optimized with the lower unit operating costs. Figure 8 shows the relationships between cost components of the transit network and unit operating costs.

The insight gained from these results is that if unit operating costs decrease, the portion of passenger travel costs in total costs increases, and the optimization process tends to reduce passenger costs more while it reduces total costs. Assuming that automation of the vehicles reduces the operating costs, it will reduce total operating costs, total costs and total passenger travel costs as well.

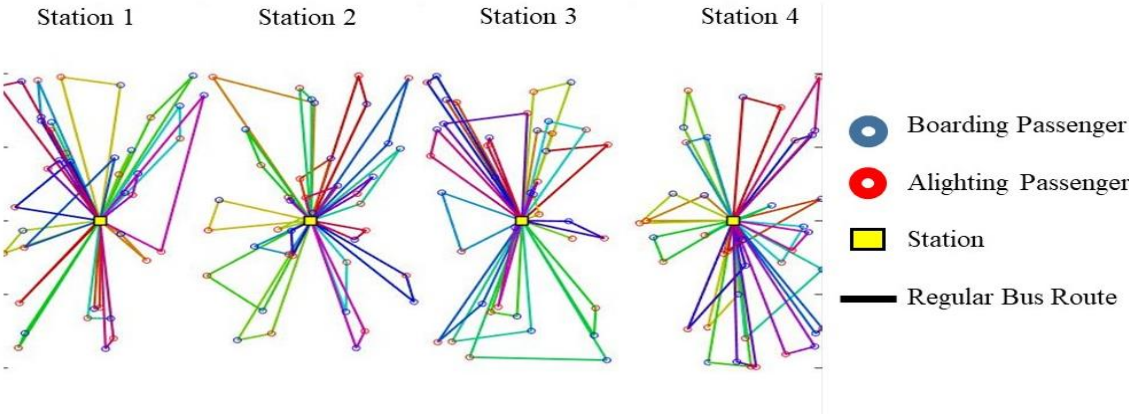


Figure 7. Results of the feeder bus routings (\$3/km Unit Operating Cost)

Table 15. The network characteristics of optimal feeder bus networks with various unit operating costs

Unit operating cost (\$/Km)	Station	Vehicle travelled Distance (Km)	Total passenger travel time (hour)	Average passenger distance travelled to each station (Km)	Total passenger average distance travelled (Km)	Total passenger travel cost (\$)	Percentage Change (passenger cost)	Total bus operating cost (\$)	Percentage Change (operating cost)	Total operating cost/Unit operating cost	Cost(\$)	Total cost(\$)	Percentage Change (total cost)	Total cost with \$5/km network
\$1/Km	#1	23.96	2.72	3.06	2.99	212.32		90.60		90.60	78.32	302.91		323.99
	#2	21.49	2.37	3.05							68.98			
	#3	23.17	2.86	3.22							80.37			
	#4	21.98	2.66	3.42							75.25			
\$2/Km	#1	20.72	2.91	3.27	3.06	217.62	2.50	172.97	90.92	86.48	99.53	390.59	28.95	403.16
	#2	20.94	2.43	3.12							90.39			
	#3	23.17	2.86	3.22							103.54			
	#4	21.66	2.69	3.46							97.13			
\$3/Km	#1	20.72	2.91	3.27	3.09	219.98	1.08	256.67	48.39	85.56	120.25	476.65	22.03	482.33
	#2	20.63	2.47	3.17							111.27			
	#3	23.03	2.87	3.23							126.57			
	#4	21.18	2.75	3.54							118.56			
\$4/Km	#1	20.00	3.03	3.40	3.21	228.28	3.77	332.40	29.50	83.10	140.49	560.68	17.63	561.50
	#2	19.53	2.65	3.41							131.21			
	#3	22.86	2.90	3.27							149.49			
	#4	20.72	2.83	3.64							139.50			
\$5/Km	#1	20.00	3.03	3.40	3.44	244.82	7.24	395.85	19.09	79.17	160.48	640.66	14.26	640.66
	#2	18.14	2.94	3.77							149.40			
	#3	22.86	2.90	3.27							172.35			
	#4	18.18	3.38	4.34							158.44			

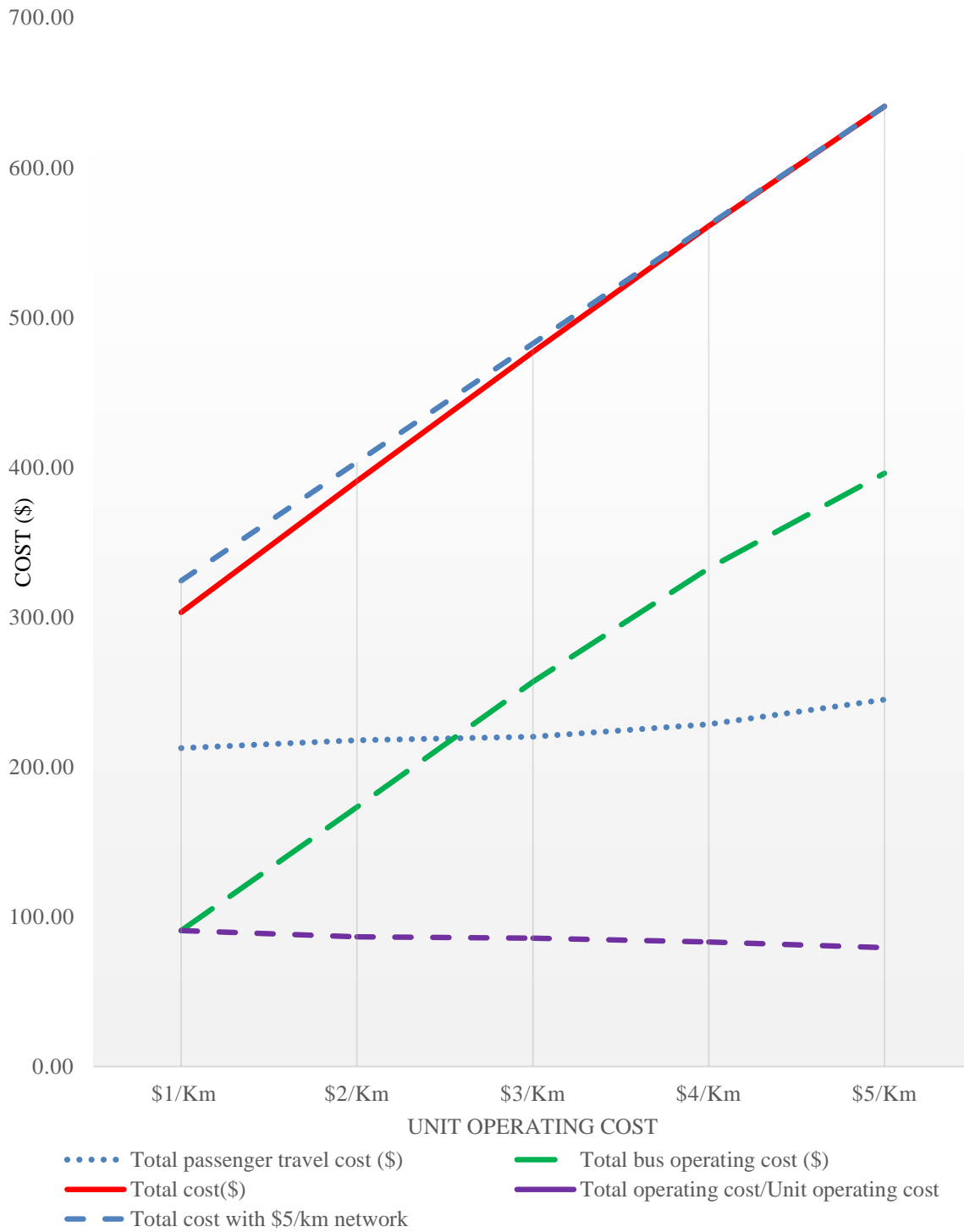


Figure 8. Various cost components for various unit operating costs

8. CONCLUSIONS

Although demand responsive feeder bus operation is possible even with human-driven vehicles, it is not very popular and mostly available as a special service because of the high operating costs due to the intensive labor costs. However, once automated vehicles become available, small-sized flexible door-to-door feeder bus operation can become more realistic, and preparing for that is necessary to catch the rapid improvement of automated vehicle technology. So, in this research, an algorithm for the optimal flexible feeder bus routing, which considers relocation of buses for multi-stations and multi-trains, was developed using an SA algorithm for future automated vehicle operation.

The example was developed and tested to demonstrate the developed algorithm. The algorithm successfully handled relocating the buses when the optimal bus routings were not feasible with the available buses at certain stations. Also the developed algorithm considered the maximum acceptable Degree of Circuity (DOC) for each passenger's trip while minimizing total costs including total passenger travel time and vehicle traveled distance. Unlike package delivery and pickup problems, each individual considers his/her travel time in the feeder bus, while a transit agency considers vehicle operating costs. In order to evaluate the impact of the acceptable maximum travel time, four types of Max *DOC* (2.5, 3, 3.5 and 4) were applied.

As expected, with higher Max *DOCs*, total costs of the service decreased due to the more relaxed constraint. With the higher ratio, the minimum number of buses used and vehicle traveled distance also decreased while passengers' total travel time more likely increased, although it did not show a clear relationship with the Max *DOC*. This study also found that the expectation of the shorter passenger travel time with the lower Max *DOC* due to a more direct route was not always fulfilled because of the less efficient routings generated by the algorithm to satisfy the lower maximum individual passenger's Degree of Circuity constraint.

Although total costs were reduced with the higher Max *DOC* due to the more relaxed constraint for the algorithm, the savings in total costs were not significant. So, transit agencies will decide whether

to choose lower individual passengers' maximum *DOC* for less maximum additional travel time for individual passengers or higher individual passengers' maximum *DOC* for less total costs and less transit operating costs.

This study also provides a mechanism for future evaluations of how efficient automated feeder services are and how they will compare with the fast-approaching automated ridesharing and carsharing services. Eventually, these studies will help predict users' travel behaviors and modal choices between the automated ridesharing/carsharing operation and the automated feeder service for mass transit.

For future research, a feeder bus routing algorithm for the trains with much shorter headways is being developed, which requires a passenger-feeder bus-train matching process in the algorithm. Also, the algorithm using smarter metaheuristics, one that incorporates composite heuristics for the larger and real networks, will be developed and adopted.

Automated vehicles are expected to provide safer service and reduce accidents, and also expected to lower operating costs by eliminating or reducing labor costs.

Unlike fixed route mass transit, small vehicle demand responsive service uses flexible routing, which means lower unit operating costs not only decrease total operating costs and total costs but also can affect routing and impact network characteristics.

In this research, optimal flexible demand responsive feeder transit networks were generated with various unit transit operating costs using the authors' previously developed routing optimization algorithm. Then network characteristics of those feeder networks were examined and compared.

The results showed that when unit operating costs decline, total operating costs and total costs obviously decline. Furthermore, when unit operating costs decline, the average passenger travel distance and total passenger travel costs decline while the ratio of total operating costs per unit operating costs increases. That means if unit operating costs decrease, the portion of passenger travel costs in total costs increases, and the optimization process tends to reduce passenger costs more while it reduces total costs. The total costs decrease with the lower unit operating costs even if the networks remain the same due to the lower total operating costs, but their total costs are always higher than the total costs of the optimized

networks because the feeder networks were not optimized with the lower unit operating costs. Assuming that automation of the vehicles reduces the operating costs, it will reduce total operating costs, total costs and total passenger travel costs as well.

In the future, more various demand distribution and scenarios will be applied to develop more general results.

9. REFERENCES

- Adewole, A., Otubamowo, K., Egunjobi, T., & Ng, K. (2012). A comparative study of simulated annealing and genetic algorithm for solving the travelling salesman problem. *International Journal of Applied Information Systems (IJ AIS)*, 4(4), 6-12.
- Ai, T. J., & Kachitvichyanukul, V. (2009). A particle swarm optimization for the vehicle routing problem with simultaneous pickup and delivery. *Computers & Operations Research*, 1693-1702.
- Almasi, M. H., Mirzapour Mounes, S., Koting, S., & Karim, M. R. (2014). Analysis of feeder bus network design and scheduling problems. *The Scientific World Journal*.
- Angelelli, E., & Mansini, R. (2002). *The vehicle routing problem with time windows and simultaneous pick-up and delivery*. In *Quantitative approaches to distribution logistics and supply chain management*. Berlin, Heidelberg: Springer.
- Avenali, A., Boitani, A., Catalano, G., D'Alfonso, T., & Matteucci, G. (2017). Assessing standard costs in local public bus transport: A hybrid cost model. *Transport Policy*. doi:<http://dx.doi.org/10.1016/j.tranpol.2017.03.011>
- Baldacci, R., Bartolini, E., & Mingozzi, A. (2011). An exact algorithm for the pickup and delivery problem with time windows. *Operations research*, 414-426.
- Batarce, M., & Galilea, P. (2018). Cost and fare estimation for the bus transit system of Santiago. *Transport Policy*. doi:<https://doi.org/10.1016/j.tranpol.2018.01.002>
- Belhaiza, S. (2017). A data driven hybrid heuristic for the dial-a-ride problem with time windows. *Computational Intelligence (SSCI)* (pp. 1-8). 2017 IEEE Symposium Series on.
- Bent, R., & Van Hentenryck, P. (2006). A two-stage hybrid algorithm for pickup and delivery vehicle routing problems with time windows. *Computers & Operations Research*, 875-893.
- Bizon, N., Dascalescu, L., & Tabatabaei, N. M. (2014). *Autonomous Vehicles: Intelligent Transport Systems and Smart Technologies*. Nova Science Publishers.
- Bosch, P. M., Becker, F., Becker, H., & Axhausen, K. W. (2017). Cost-based analysis of autonomous mobility services. *Transport Policy*. doi:<https://doi.org/10.1016/j.tranpol.2017.09.005>
- Bräysy, O., & Gendreau, M. (2005). Vehicle routing problem with time windows, Part II: Metaheuristics. *Transportation science*, 39(1), 119-139.
- Bureau of Labor Statistics (BLS). (2018). *CPI Inflation Calculator*. Retrieved July 25, 2018, from Bureau of Labor Statistics: https://www.bls.gov/data/inflation_calculator.htm
- Chien, S., & Yang, Z. (2000). Optimal feeder bus routes on irregular street networks. *Journal of Advanced Transportation*, 34(2), 213-248.
- Chien, S., Yang, Z., & Hou, E. (2001). Genetic algorithm approach for transit route planning and design. *Journal of transportation engineering*, 127(3), 200-207.
- Chong, Z. J., Qin, B., Bandyopadhyay, T., Wongpiromsarn, T., Rankin, E. S., Ang Jr., M. H., . . . Low, K. H. (2011). Autonomous Personal Vehicle for the First- and Last-Mile Transportation Services.

- 2011 IEEE 5th International Conference on Cybernetics and Intelligent Systems (CIS) (pp. 253-260). Institute of Electrical and Electronics Engineers.
- Ciaffi, F., Cipriani, E., & Petrelli, M. (2012). Feeder bus network design problem: A new metaheuristic procedure and real size applications. *Procedia-Social and Behavioral Sciences*, 54, 798-807.
- Concas, S., Winters, P., & Wambalaba, F. (2005). Fare pricing elasticity, subsidies, and demand for vanpool services. *Transportation Research Record: Journal of the Transportation Research Board*, 215-223.
- Cordeau, J. F., & Laporte, G. (2003). The dial-a-ride problem (DARP): Variants, modeling issues and algorithms. *Quarterly Journal of the Belgian, French and Italian Operations Research Societies*, 89-101.
- Cordeau, J. F., & Laporte, G. (2007). The dial-a-ride problem: models and algorithms. *Annals of operations research*, 153(1), 29-46.
- Cubillos, C., Urra, E., & Rodríguez, N. (2009). Application of genetic algorithms for the DARPTW problem. *International Journal of Computers Communications & Control*, 127-136.
- Davendra, D., Zelinka, I., & Onwubolu, G. (2010). Chaotic attributes and permutative optimization. *Evolutionary Algorithms and Chaotic Systems*, 481-517.
- Deng, L. B., Gao, W., Zhou, W. L., & Lai, T. Z. (2013). Optimal design of feeder-bus network related to urban rail line based on transfer system. *Procedia-Social and Behavioral Sciences*, 2383-2394.
- Department for Transport (DfT). (2017). *Bus statistics*. Retrieved July 25, 2018, from GOV.UK: <https://www.gov.uk/government/collections/bus-statistics>
- Dumas, Y., Desrosiers, J., & Soumis, F. (1991). The pickup and delivery problem with time windows. *European journal of operational research*, 7-22.
- Fazlollahtabar, H., & Saidi-Mehrabad, M. (2015). *Autonomous Guided Vehicles: Methods and Models for Optimal Path Planning*. Springer International Publishing. doi:10.1007/978-3-319-14747-5
- Federal Transit Administration (FTA). (2018, July 24). Retrieved from The National Transit Database (NTD): <https://www.transit.dot.gov/ntd>
- Furuhata, M., Dessouky, M., Ordóñez, F., Brunet, M. E., Wang, X., & Koenig, S. (2013). Ridesharing: The state-of-the-art and future directions. *ransportation Research Part B: Methodological*, 28-46.
- Gajpal, Y., & Abad, P. (2009). An ant colony system (ACS) for vehicle routing problem with simultaneous delivery and pickup. *Computers & Operations Research*, 3215-3223.
- Gholami, A., & Mohaymany, A. S. (2011). Economic conditions for minibus usage in a multimodal feeder network. *Transportation Planning and Technology*, 34(8), 839-856.
- Hasija, S., & Rajendran, C. (2004). Scheduling in flowshops to minimize total tardiness of jobs. *International Journal of Production Research*, 42(11), 2289-2301.
- Huang, H. J., Yang, H., & Bell, M. G. (2000). The models and economics of carpools. *The annals of regional science*, 55-68.

- Kaan, L., & Olinick, E. V. (2013). The vanpool assignment problem: Optimization models and solution algorithms. *Computers & Industrial Engineering*, 24-40.
- Karamanis, R., Niknejad, A., & Angeloudis, P. (2017). A fleet sizing algorithm for autonomous car sharing. *Transportation Research Board 96th Annual Meeting*. Washington DC, United States.
- Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *science*, 220(4598), 671-680.
- Krueger, R., Rashidi, T. H., & Rose, J. M. (2016). Preferences for shared autonomous vehicles. *Transportation research part C: emerging technologies*, 69, 343-355.
- Kuah, G. K., & Perl, J. (1988). Optimization of feeder bus routes and bus-stop spacing. *Journal of Transportation Engineering*, 114(3), 341-354.
- Kuah, G. K., & Perl, J. (1988). Optimization of feeder bus routes and bus-stop spacing. *Journal of Transportation Engineering*, 114(3), 341-354.
- Kuan, S. N., Ong, H. L., & Ng, K. M. (2006). Solving the feeder bus network design problem by genetic algorithms and ant colony optimization. *Advances in Engineering Software*, 37(6), 351-359.
- Kuan, S., Ong, H., & Ng, K. (2004). Applying metaheuristics to feeder bus network design problem. *sia-Pacific Journal of Operational Research*, 21(4), 543-560.
- Lee, Y. J. (2012). Comparative measures for transit network performance analysis. *Journal of the Transportation Research Forum*, 47(3).
- Lee, Y. J., Choi, J. Y., Yu, J. W., & Choi, K. (2015). Geographical applications of performance measures for transit network directness. *Journal of Public Transportation*, 18(2), 7.
- Lee, Y.-J., Meskar, M., Nickkar, A., & Sahebi, S. (2018). Developing an Algorithm for the Optimal Flexible Automated Feeder Transit Network. *Transportation Research Board 97th Annual Meeting*. Washington DC, United States.
- Li, Z. C., Lam, W. H., & Wong, S. C. (2009). Optimization of a Bus and Rail Transit System with Feeder Bus Services under Different Market Regimes. *Transportation and Traffic Theory 2009: Golden Jubilee pp 495-516*. Springer, Boston, MA.
- Lin, J. J., & Wong, H. I. (2014). Optimization of a feeder-bus route design by using a multiobjective programming approach. *Transportation Planning and Technology*, 37(5), 430-449.
- Litman, T. (2018). *Autonomous Vehicle Implementation Predictions Implications for Transport Planning*. Victoria: Victoria Transport Policy Institute.
- Martins, C. L., & Pato, M. V. (1998). Search strategies for the feeder bus network design problem. *European Journal of Operational Research*, 106(2-3), 425-440.
- Min, H. (1989). The multiple vehicle routing problem with simultaneous delivery and pick-up points. *Transportation Research Part A: General*, 377-386.
- Mohaymany, A. S., & Gholami, A. (2010). Multimodal Feeder Network Design Problem: Ant Colony Optimization Approach. *Journal of Transportation Engineering*, 136(4), 323-331.

- Molenbruch, Y., Braekers, K., & Caris, A. (2017). Typology and literature review for dial-a-ride problems. *Annals of Operations Research*, 295-325.
- Montané, F. A., & Galvao, R. D. (2006). A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service. *Computers & Operations Research*, 595-619.
- Montoya-Torres, J., Franco, Julián López, Nieto Isaza, S., Jiménez, H. F., & Herazo-Padilla, N. (2015). A literature review on the vehicle routing problem with multiple depots. *Computers & Industrial Engineering*, 79, 115-129.
- Nagata, Y., & Kobayashi, S. (2010). Guided ejection search for the pickup and delivery problem with time windows. *European Conference on Evolutionary Computation in Combinatorial Optimization* (pp. 202-213). Berlin, Heidelberg: Springer.
- Nanry, W. P., & Barnes, J. W. (2000). Solving the pickup and delivery problem with time windows using reactive tabu search. *Transportation Research Part B: Methodological*, 107-121.
- On Target Performance Group. (2011). *Evaluation: Bus Cost Allocation Model Regional Transportation District Denver, Colorado*.
- Potvin, J. Y. (2009). A review of bio-inspired algorithms for vehicle routing. *Bio-inspired algorithms for the vehicle routing problem*, 1-34.
- Quarles, N., & Kockelman, K. M. (2018). Costs and Benefits of Electrifying and Automating U.S. Bus Fleets. *Transportation Research Board 97th Annual Meeting*. Washington, D.C.: Transportation Research Board (TRB).
- Reich, S. L., & Davis, J. L. (2011). *Analysis of Contracting for Fixed Route Bus Service*. Tampa, Florida: National Center for Transit Research.
- Rochat, Y., & Taillard, É. D. (1995). Probabilistic diversification and intensification in local search for vehicle routing. *Journal of heuristics*, 1(1), 147-167.
- Ropke, S., & Pisinger, D. (2006). An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation science*, 455-472.
- Samól, D. (2017). *Integrated Infrastructures and Processes Across Energy, ICT and Transport*. In *Happy City-How to Plan and Create the Best Livable Area for the People*. Cham: Springer.
- Shin, H.-S., Callow, M., Dadvar, S., Lee, Y.-J. & Farkas, Z.A., (2015). User Acceptance and Willingness to Pay for Connected Vehicle Technologies: Adaptive Choice-Based Conjoint Analysis. *Transportation Research Record: Journal of the Transportation Research Board*, 2531, 54-62.
- Shin, H.-S., Callow, M., Farkas, Z. A. & Lee, Y.-J. (2016). *Measuring User Acceptance of and Willingness-to-Pay for CVI Technology*. Connected Vehicle/Infrastructure University Transportation Center (CVI-UTC).
- Shrivastav, P., & Dhingra, S. L. (2001). Development of feeder routes for suburban railway stations using heuristic approach. *Journal of transportation engineering*, 127(4), 334-341.
- Shrivastava, P., & O'Mahony, M. (2006). A model for development of optimized feeder routes and coordinated schedules—A genetic algorithms approach. *Transport policy*, 13(5), 413-425.

- Shrivastava, P., & O'Mahony, M. (2007). Design of feeder route network using combined genetic algorithm and specialized repair heuristic. *Journal of public transportation*, 10(2), 7.
- Shrivastava, P., & O'Mahony, M. (2009). Use of a Hybrid Algorithm for Modeling Coordinated Feeder Bus Route Network at Suburban Railway Station. *Journal of Transportation Engineering*, 135(1), 1-8.
- Souza, M. J., Mine, M. T., Silva, M. D., Ochi, L. S., & Subramanian, A. (2011). A hybrid heuristic, based on Iterated Local Search and GENIUS, for the Vehicle Routing Problem with Simultaneous Pickup and Delivery. *International Journal of Logistics Systems and Management*, 142-157.
- Stocker, A., & Shaheen, S. (2017). *Berkeley Shared Automated Vehicles: Review of Business Models*. International Transport Forum (ITF).
- Sun, P., Veelenturf, L. P., Hewitt, M., & Van Woensel, T. (2018). The time-dependent pickup and delivery problem with time windows. *Transportation Research Part B: Methodological*, 1-24.
- Tasan, A. S., & Gen, M. (2012). A genetic algorithm based approach to vehicle routing problem with simultaneous pick-up and deliveries. *Computers & Industrial Engineering*, 755-761.
- Tripathy, T., Nagavarapu, S. C., Azizian, K., Pandi, R. R., & Dauwels, J. (2017). Solving Dial-A-Ride Problems Using Multiple Ant Colony System with Fleet Size Minimisation. *UK Workshop on Computational Intelligence* (pp. 325-336). Cham: Springer.
- Ungemah, D., Rivers, M., & Anderson, S. (2006). The world can be flat: Case study of flat-rate pricing for vanpool operations. *Transportation research record*, 30-33.
- Van Breedam, A. (1996). An analysis of the effect of local improvement operators in genetic algorithms and simulated annealing for the vehicle routing problem. *RUCA*.
- Victoria Transport Policy Institute. (2016). Vehicle Costs. In *Transportation Cost and Benefit Analysis Techniques, Estimates and Implications* (2nd ed., pp. 5.1-16). Victoria, British Columbia, Canada. Retrieved July 25, 2018, from <http://www.vtppi.org/tca/tca0501.pdf>
- Wang, C., Mu, D., Zhao, F., & Sutherland, J. W. (2015). A parallel simulated annealing method for the vehicle routing problem with simultaneous pickup-delivery and time windows. *Computers & Industrial Engineering*, 83, 111-122.
- Washbrook, K., Haider, W., & Jaccard, M. (2006). Estimating commuter mode choice: A discrete choice analysis of the impact of road pricing and parking charges. *Transportation*, 621-639.
- Yan, S., & Chen, C. Y. (2011). A model and a solution algorithm for the car pooling problem with pre-matching information. *Computers & Industrial Engineering*, 512-524.
- Zhang, W., Jenelius, E., & Badia, H. (2018). Bus operation modeling for comparing conventional, semi-autonomous and fully autonomous bus services. *Preprint*.
- Zidi, I., Zidi, K., Mesghouni, K., & Ghedira, K. (2011). A multi-agent system based on the multi-objective simulated annealing algorithm for the static dial a ride problem. *18th World Congress of the International Federation of Automatic Control (IFAC)*. Milan, Italy.